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Simultaneous optimization of radar waveform and mismatched filter with range and delay-Doppler sidelobes suppression

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ABSTRACT

Since radar transmit circuit often operates in the saturation mode to maximize transmit power, radar is more suitable for transmitting constant modulus waveforms, which makes it difficult to suppress the range sidelobes. The mismatched filter method in the receive end can suppress range sidelobes significantly, but it is often designed with a prescribed waveform. Hence, in this paper, we firstly present an approach to simultaneously optimize radar waveform and mismatched filter (SORW-MF) under a signal-to-noise ratio (SNR) loss constraint. However, most of existing mismatched filter design works including the aforementioned SORW-MF work neglect Doppler mismatch issue. To address this problem, during SORW-MF process, another approach is developed to suppress delay-Doppler sidelobes and control SNR loss. Both design approaches are non-convex optimization problems, and we solve them by a least-*p*th minimax algorithm and a double least-*p*th minimax algorithm, respectively. Numerical results indicate that the first approach can further reduce the range sidelobes compared with the separate/iterative design methods, and the second approach can efficiently suppress delay-Doppler sidelobes over different specifications of Doppler mismatch.

1. Introduction

Radar waveform is the information carrier for a radar system and waveform optimization is a critical issue in the radar field (see [1–8] and the references therein). For high time-bandwidth waveforms, pulse compression is often necessary to achieve good range resolution and target detection capability, which inevitably produces undesirable outputs at shifted range bins (i.e., range sidelobes). A high range sidelobe level may trigger false alarms or cause high-power target returns submerging low-power target returns in range bins nearby. Consequently, for pulse compression radar, an interesting issue is to suppress range sidelobes through waveform optimization.

In practice, radar amplifiers often work in saturation mode to maximize its efficiency. Hence, the transmit waveform should be constant modulus. Phase coded waveform (PCW) is a typical constant modulus signal and its design has been extensively studied, including binary codes [1], polyphase codes [2] and continuous phase codes [3–8]. For LFM waveforms, the windowing functions are often used in range compression for low range sidelobes. For PCWs, to further suppress range sidelobes, a typical method is to design a mismatched filter with a given waveform [9–12], which belongs to a convex optimization problem and can be conveniently solved [9,10]. But the final sidelobe levels of mismatched filter output depend on the characteristic of the given transmit waveform. Therefore, we can jointly optimize a transmit waveform and its corresponding mismatched filter for a better performance [13–23]. In [13], an exhaustive searching method is proposed to suppress peak sidelobe level (PSL) and integrated sidelobe level (ISL), but the computational burden is heavy. In [14], at the background of colocated MIMO radar, auto-correlation and cross-correlation sidelobes at/between given spacial angles are suppressed by alternately optimizing transmit waveforms and receive filter bank. In [15], joint design is performed in the frequency domain and a synthesis step is used to obtain a temporal waveform and a mismatched filter. In [16,17], the authors attempt to maximize signal-to-interference-plus-noise ratio (SINR) by iteratively optimizing transmit waveform (slow-time) and receive filter, where a similar constraint and an energy constraint are imposed on the transmit waveform. A framework similar to that of [16] is concerned in [18] but transmit waveform is constant modulus. In [19], the authors deal with the problem of intrapulse radar-embedded communication through maximizing signal-to-interference ratio and minimizing the probability of intercept. In [20–22], transmit waveforms

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and its mismatched filter are alternatively optimized to minimize mean-square error (MSE) of estimation of scattering coefficients of point-like targets. In [19–22], for fixed waveform, a filter can be written as a closed-form expression by minimizing a SIR, ISL or MSE defined therein. In [23,24], the authors consider to iteratively optimize a transmit waveform and a filter bank by maximizing the worst SINR of filter bank output with respect to target aspect angle, but the transmit waveform is not constant modulus. Most of these works mentioned above use iterative optimization approaches for joint design.

In this paper, we firstly focus on simultaneously optimizing a transmit waveform and a mismatched filter to suppress range sidelobes of filter output. A min–max criterion is presented to suppress PSL and control signal-to-noise ratio (SNR) loss, which is a non-convex and constrained optimization problem. To tackle it, we firstly utilize a penalty method to transform the optimization problem into an unconstrained optimization problem. Then, we exploit a least-pth minimax (LPM) algorithm with a Limited-Memory Broyden–Fletcher–Goldfarb and Shannon (L-BFGS) [26] algorithm as its subalgorithm to approximately address it. Numerical results are given and indicate that this approach outperforms the matched filter, the mismatched filter with well-optimized constant modulus waveforms and alternative optimization method (measured by range sidelobes).

We further point out that most of existing works do not take the Doppler mismatch into account, which is often inevitable in practice. In presence of slight Doppler mismatch, the range sidelobes would rise significantly. Therefore, mismatched filter design should take the Doppler issue into consideration in most situations. In [27], mismatched filter with better Doppler tolerance is optimized with a prescribed constant modulus waveform. In [28,29], the iterative optimization method is used to jointly design receive filter and transmit sequence (slow-time) with amplitude and phase modulations, and its method aims at improving the worst SINR over certain Doppler range. In [30], the Doppler issue is concerned in joint design and the purpose is to minimize MSE of the estimation of target scattering coefficients instead of suppressing range sidelobes in presence of signal-dependent clutter and signal-independent interference environment, which needs know the range bin of targets. Simultaneous optimization of transmit waveform and mismatched filter with Doppler concern is seldom concerned in existing works. Therefore, in this paper, we provide another simultaneous optimization approach with Doppler concern. Specifically, we propose a minimization criterion to simultaneously optimize a PCW and a mismatched filer (SOPCW-MF) for low delay-Doppler sidelobes (measured by PSL) and a low SNR loss, which is also a non-convex and unconstrained optimization problem. We propose a double least-pth minimax (DLPM) algorithm to optimize it, which stems from the LPM algorithm.

Note that we just suppress the delay-Doppler sidelobes over a specific Doppler range, instead of suppressing whole Doppler frequencies area. Because in the searching mode, we often have certain knowledge about the maximum radial velocities of targets of interest, and in the tracking mode, some samples can be used to estimate and predict the radial velocity information of a target. In either case, a range of Doppler frequencies can be roughly obtained. Therefore, keeping Doppler robustness over the interested region is a more efficient way and there is a parameter to control the range of Doppler frequencies. Numerical results are presented to show optimization results with different Doppler ranges. It is found that firstly, Doppler mismatch causes significant increase of range sidelobes and secondly, the simultaneous optimization method can successfully control the delay-Doppler sidelobes over a prescribed Doppler range. Our proposed approaches can be used in radar systems using phased-coded signals expecting low sidelobes even with slight Doppler mismatch.

The rest of this paper is organized as follows. In Section 2, the problem formulation and optimization method of SOPCW-MF with range sidelobes suppression are presented. The problem formulation and optimization method of SOPCW-MF with Doppler concern are provided in Section 3. In Section 4, simulation results are presented. Finally, conclusions are drawn in Section 5.

2. SOPCW-MF with range sidelobes suppression

2.1. Problem formulation

Consider a radar system that transmits a PCW, denoted by $\mathbf{s}_1 \in \mathbb{C}^{N_s}$, where N_s is the code length of PCW, $\mathbb{C}^{N_s}$ denotes N_s dimensional complex vector space. Let $\mathbf{h}_1 \in \mathbb{C}^{N_h}$ $(N_h \geq N_s)$ be the corresponding mismatched filter, where N_h is the length of mismatched filter. The output of PCW $\mathbf{s}_1$ after mismatched filtering $\mathbf{h}_1$ at range shift k can be written as

$$\omega(k) = \sum_{n=-\infty}^{\infty} \mathbf{s}_1(n)\mathbf{h}_1^*(n-k) \tag{1}$$

where $(\cdot)^*$ denotes the conjugate operation. The nth component of PCW $\mathbf{s}_1$ and mismatched filter $\mathbf{h}_1$ are

$$\mathbf{s}_1(n) = \begin{cases} e^{j\varphi_1(n)}, & \text{for } n \in [1, N_s] \\ 0, & \text{others} \end{cases} \tag{2}$$

and

$$\mathbf{h}_1(n) = \begin{cases} \mathbf{a}_1(n)e^{j\boldsymbol{\theta}_1(n)}, & \text{for } n \in [1, N_h] \\ 0, & \text{others} \end{cases} \tag{3}$$

respectively, where $\boldsymbol{\varphi}_1$ is the phase vector of PCW $\mathbf{s}_1$, i.e., $\mathbf{s}_1 = \exp(j\boldsymbol{\varphi}_1)$, $\mathbf{a}_1$ and $\boldsymbol{\theta}_1$ are the magnitude vector and phase vector of mismatched filter $\mathbf{h}_1$, respectively, i.e., $\mathbf{h}_1 = \mathbf{a}_1 \odot \exp(j\boldsymbol{\theta}_1)$, $j = \sqrt{-1}$, and $\odot$ denotes the Hadamard product. Two properties are considered in SOPCW-MF:

1) Range sidelobes suppression: In practice, the range sidelobes of mismatched filter output should be low as much as possible so that high-power target returns would not submerge low-power target returns in nearby range bins. In addition, low PSL can prevent high-power target returns triggering false alarms through range sidelobes. We would measure the range sidelobes through the PSL. Specifically, to simplify the notation, define $\Gamma_1 = [-(N_h + N_s)/2 + 1, \ldots, -1, 1, \ldots, (N_h + N_s)/2 - 1]$. Stack all the range sidelobes in (1) into a vector $\boldsymbol{\omega}_{\text{side}}$ $(\forall k \in \Gamma_1)$. A min–max optimization criterion aiming at suppressing PSL is

$$\min_{\boldsymbol{\varphi}_1, \mathbf{a}_1, \boldsymbol{\theta}_1} \max_{k \in \Gamma_1} |\boldsymbol{\omega}_{\text{side}}|. \tag{4}$$

However, it can be proved that a solution is that $\mathbf{h}_1$ or $\mathbf{a}_1$ is an all-zero vector. To make the result meaningful, we need to consider another constraint.

2) SNR loss control: Mismatched filter would inevitably lead to SNR loss, which is undesirable. If $\mathbf{a}_1 = \mathbf{0}$, no SNR would be left, which is not a good choice. To control the SNR loss within an acceptable level, we propose a simple but efficient approach to control SNR loss.

The SNR loss of PCW $\mathbf{s}_1$ after mismatched filtering $\mathbf{h}_1$ (usually $\mathbf{h}_1^{\text{H}}\mathbf{h}_1 \geq N_s$) can be expressed by [31]

$$\text{SNR}_{\text{loss}} = 10 \times \log_{10}\left(N_s \mathbf{h}_1^{\text{H}}\mathbf{h}_1 / \omega^*(0)\omega(0)\right). \tag{5}$$

where $(\cdot)^{\text{H}}$ denotes the conjugate transpose operation.

If we design mismatched filter with a prescribed waveform, a typical constraint is $\omega(0) = N_s$ $(\mathbf{s}_1^{\text{H}}\mathbf{s}_1 = N_s)$ [9], and the SNR loss is simplified into