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On the flexibility of a decision theory-based heuristic for single machine scheduling

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ABSTRACT

This report extends prior research on the “decision theory” approach for scheduling/sequencing (DTS). Compared to other construction heuristics like priority dispatching (PD) approaches, DTS has the advantage that it is flexible regarding a diverse range of regular and non-regular objectives. Furthermore, multiple decision criteria linearly combined within a single objective function can be addressed.

For sequencing a set of jobs on a single machine, DTS estimates the total effect of selecting the next job in the sequence. To this, the completion times for all jobs resulting from this decision need to be estimated. We provide an estimator for job completion times and prove it to be the expected completion time. We also prove that DTS using this estimator provides optimum solutions for a number of single machine scheduling problems. Finally, we provide an extensive computational study comparing DTS to 38 competing PD approaches for a large variety of objectives (31). The results indicate DTS to be a flexible and viable alternative to PD approaches almost independent of specific objectives and problem instance characteristics.

1. Introduction

For more than four decades, the development and use of priority dispatching (PD) rules have played a prominent role in both scheduling theory and the practice of industrial scheduling. The PD approach is easy to understand, simple to apply, and in many cases yields good solutions. However, PD rules suffer from the defect of being aimed at a specific objective function. One rule might perform well when minimizing flow time; another rule might work best when minimizing tardiness is the criterion. As pointed out by Kanet and Zhou (1993), a second drawback of traditional PD is its inherent myopic view when selecting the next job to dispatch. All PD approaches determine a measure of urgency (a priority index) for each job and select the most urgent job (with the highest priority) to be dispatched. The inherent opportunity cost that all other jobs will not be selected is not considered. To overcome this drawback, Chryssolouris et al. (1988, 1991) developed a scheduling approach, which has become known as the “decision theory approach” (DT) for sequencing. The DT approach is based on estimating the (full) consequences (on the objective function value) of selecting a given job to be dispatched next. Thereby, the consequence of selecting one job next includes not only its effect on the objective function value but also the expected effects of all other jobs that are not chosen next. In consequence, the job that provides the most favorable expected “total” consequence is dispatched next (Kanet and Zhou, 1993).

In this paper, we further develop the idea of DT sequencing (DTS). To confirm the worthiness of DTS, we limit the analysis here to a systematic study of single machine sequencing problems in which the objective is the minimization of different functions f on the completion times of a set of jobs ($f(C)$). Beside regular functions (f_{reg}) like total flowtime or total tardiness, we also consider non-regular objective functions f_{nreg} with the restriction that no idle time is allowed. Generally, only non-delay sequences are considered. This restriction to non-delay sequences, even for non-regular objectives, is very common for different single machine sequencing objectives like the minimization of “total weighted earliness and tardiness” (see e.g., Ow and Morton, 1989; Valente and Alves, 2005), “total weighted quadratic earliness and tardiness” (Vila and Pereira, 2013), or “completion time variance” (Srirangacharyulu and Srinivasan, 2010). Focusing on single machine problems is not as restrictive as it may appear because many variants of the problem types $1||f_{reg}(C_j)$ and $1||f_{nreg}(C_j)$, though well studied, are still of theoretical and practical importance. The attention to such single machine problems is well justified as it applies to many industrial settings, e.g., paint shops in a car manufacturing

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facility (Bock and Pinedo, 2010) or any serial production facility or assembly line that is scheduled as a single entity (Pinedo, 2009). Furthermore, with the long-standing industrial focus to move from job-shop process design to work cells, one sees work cells scheduled as a single entity (Wemmerlöv and Hyer, 1989). In addition, for many more scheduling environments, like parallel machines, findings for the $1||f(C_j)$ problem can be used to optimize the sequence on each machine more efficiently (Koulamas, 2010).

Generally, we extend the work of Kanet and Zhou (1993) who demonstrated the viability of the DT approach in a small experimental study. We extend that study in several ways:

- We prove the completion time estimator used here is the expected value.
- We prove for some special cases that DTS yields optimum sequences.
- In our experimental study, we consider exhaustive sets of objective criteria (regular and non-regular) and competitive PD approaches.
- For weighted objective criteria, we carefully compare DTS to PD approaches under different assumptions to the nature of the weights (arbitrary (unrestricted), agreeable, and proportional weights).

The remainder of this paper is organized as follows. In Section 2 we specify how the DT approach can be implemented for single machine sequencing problems along with the aforementioned proofs of the unchosen jobs' completion time estimator and the special cases. Section 3 follows with a description of the experimental study conducted to compare DTS to competing PD approaches. Here, we concentrate our analysis on "conventional" PD approaches, i.e., single-pass construction heuristics. For each considered objective, we searched the literature for the best published approach with which we could compare DTS. Section 4 reports experimental results. Section 5 summarizes and comments on future research directions.

2. Decision theory-based single machine sequencing

Formally, the scope of the problems we address here is as follows. A set N of $n=|N|$ independent jobs (indexed by $j=1, 2, \ldots, n$) has to be sequenced on a single machine that can process at most one job at a time. Preemption of jobs is not allowed and the machine is continuously available. Each job j has a ready time equal to zero, a processing time p_j, a due date d_j, a weight (tardiness penalty factor) w_j, and an earliness penalty factor h_j.. Based on the completion time C_j of job j, we compare the performance of DTS to PD approaches over a variety of regular objective functions $f_{reg}(C_1, C_2, \ldots, C_n)$ and non-regular objective functions $f_{nreg}(C_1, C_2, \ldots, C_n)$.

Generally, the literature on decision theory based sequencing or scheduling at all is quite rare. As stated above, the basic idea of DTS is attributable to Chryssolouris et al. (1988; 1991) and their MADEMA (MAnufacturing DEcision MAking) framework. Based on MADEMA, Kanet and Zhou (1993) explicitly formulate the DT approach for scheduling problems. Sridharan and Zhou (1996a;b) extend the previous work by considering release dates and the objectives total tardiness minimization and weighted earliness and tardiness minimization, respectively. Mönch et al. (2005) address the problem to schedule jobs on parallel batch machines with incompatible job families and unequal release dates. Within their genetic algorithm, they solve $1|r_j, batch, incompatible|\Sigma w_j T_j$ subproblems via DT. Their results show that the DT approach outperforms all other heuristic methods in terms of solution quality but at costs of higher computation times. DTS might also be seen as a simplification of the "Beam Search" approach of Ow and Morton (1989). Limiting the "beamwidth" parameter to one leads to DTS. For the application of Beam Search see Morton and Pentico (1993) and Sabuncuoglu and Bayiz (1999). Following these authors, the general course of action of DTS is as follows (N_t defines the set of jobs yet to be sequenced at time (decision point) t; $n_t=|N_t|$):

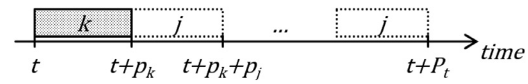

Fig. 1. Illustration of estimated job completion times.

Procedure DTS

Step 0:	Set $t=0$ and $N_t=N$.
Step 1:	For each job k from N_t tentatively select k to start at time t and compute $C_k=t+p_k$.
Step 1.1:	For each other job $j \in N_t$ ($j \neq k$), estimate its completion time C_j given that job k starts at time t (via Eq. (1); see below).
Step 1.2:	Compute the estimated objective value $Z_k=f(C_1,\ldots, C_n)$.
Step 2:	Choose that job k^* to be sequenced next with smallest estimated Z_k.
Step 3:	Set $t=t+p_{k^*}$ and remove job k^* from N_t.
Step 4:	If N_t is not empty, then go to Step 1.

As can be seen by procedure DTS, the algorithm complexity is $O(n^3)$, which is somewhat higher compared to the complexities of static PD approaches (with $O(n \log n)$) and most dynamic PD approaches (with $O(n^2)$). It should be noted that some more sophisticated dynamic PD approaches also have a complexity of $O(n^3)$. Nevertheless, there are several major advantages of DTS over many PD approaches:

- First, the logic of DTS is easy to follow. It contains no complex formulas as in some PD approaches.
- Second, no a priori parameter estimation is required (e.g., look-ahead parameters or job allowance factors).
- Third, DTS takes a global view in selecting a job to be sequenced next (global in the sense that it considers the opportunity cost of not selecting other jobs), which sets it apart from "myopic" PD approaches that considers only the attributes of each job individually (see e.g., Sridharan and Zhou, 1996a).
- Fourth, and most importantly, it is flexible with regard to the objective criterion. Except for changing the objective function, no other changes regarding the implementation are required.

2.1. Estimating job completion times for DTS

To apply DTS, we need to estimate job completion times as required in Step 1.1 of Procedure DTS shown above. To estimate these completion times given that some job k is scheduled next, we use the following estimator of a job j's completion time $\tilde{E}(C_j)$ as used by Sridharan and Zhou (1996a;b):

$$\tilde{E}(C_j) = t + \frac{p_k + p_j + P_t}{2} \quad \forall j \in N_t, \ \ j \neq k. \tag{1}$$

In expression (1), P_t refers to the total processing time of all jobs $j \in N_t$ to be sequenced at this point in time ($P_t = \sum_{j\in N_t} p_j$). At time t, we tentatively select job k to be sequenced and thus, any other unselected job j will complete processing somewhere in the interval $[t+p_k+p_j, t+P_t]$ (Fig. 1 illustrates).

We prove in the following that this estimator (expression (1)) is the expected value of a job's completion time. Note that when a job k is chosen, then each remaining unchosen job j can, with equal probability, occupy any one of the $u=1, \ldots, n_t-1$ (with $n_t=|N_t|$) positions after job k. Given any position u for job j, there are $\binom{n_t-2}{u-1}$ ways that $u-1$ of the remaining n_t-2 jobs can be chosen to reside in the schedule between k and j. Using $\sum_{a=0}^{b}\binom{b}{a} = 2^b$, the total number of outcomes (the event space) for all positions