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A generalized flow splitting model for day-to-day traffic assignment

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Abstract

The splitting rate model proposed by Smith and Mounce (2011) establishes a traffic evolution process on a link-node network representation, which overcomes the difficulties in applying traditional path-based models and provides the ease of implementing controls at nodes. While their model offers a new method for modeling traffic evolution, it contains an ad-hoc step of flow adjustment to preserve the flow conservation. This flow adjustment step leads to difficulties in analyzing the system properties. This paper proposes a generalized flow splitting model for day-to-day traffic assignment based on the concept of splitting flow at nodes. The proposed model preserves the flow conservation endogenously by introducing the inflow variable into the formulation. The generalized formulation provides the ease to construct a variety of day-to-day traffic assignment models, and serves as a framework for analyzing the models' properties, such as the invariance property and the preservation of the Lipschitz continuity and strong monotonicity. Specifically, a proportional-adjustment model and a projection-type model are developed based on the proposed generalized formulation. A numerical example demonstrates the ease of implementing the proposed generalized model, as well as its convergence to user equilibrium.

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Keywords: day-to-day traffic assignment; flow splitting; flow conservation; user equilibrium.

1. Introduction

With the advancement of intelligent transportation systems, travelers have the capability of accessing historical and real-time traffic information, thereby adjusting their path choices in accordance with their day-to-day experience and information provision. Understanding and modeling travelers' day-to-day choice adjustment plays an important

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role in developing effective advance traffic management systems. Many day-to-day traffic assignment models have been proposed and used in day-to-day traffic control strategies such as dynamic congestion pricing (Friesz et al. 2004; Yang and Szeto 2006; Yang et al. 2007; Guo 2013; Guo et al. 2013), signal timing (Hu and Mahmassani 1997; Smith and Mounce 2011), and transit operator strategies (Cantarella et al. 2013).

In the literature, most deterministic day-to-day traffic assignment models are path-based models, i.e., they all explicitly use path flow variables and provide explicit path flow evolution trajectories. Yang and Zhang (2009) classify these path-based models into four categories: the proportional-switch adjustment process (Smith, 1984; Smith and Wisten 1995; Peeta and Yang 2003), network tâtonnement adjustment (Friesz et al. 1994), projected dynamical system (Zhang and Nagurney 1996; Nagurney and Zhang 1997) and evolutionary traffic dynamics (Sandholm 2001; Yang 2005). Since these models involve a differential equation for each possible path, the problem becomes intractable when the network size increases.

He et al. (2010) discuss the shortcomings of path-based models and propose a model built upon link flows to eliminate the realism issues inherent in path-based models. Since its state variables contain link flows only, the link-based model (LBM) has a tractable problem size that is suitable for analyzing the traffic evolution for a large-scale network, e.g., the traffic evolution after the I-35W Bridge collapse in Minneapolis, Minnesota (He and Liu 2012). Han and Du (2012) extend the LBM to model traffic evolution for networks with non-separable link cost functions. Guo et al. (2013) develop a generalized link-based model to cover a variety of discrete-time day-to-day traffic evolution processes.

Another way to overcome the shortcomings in path-based models is to construct a traffic evolution model on a link-node network representation. Smith and Mounce (2011) propose a splitting rate model (SRM), which adjusts link flow splitting rates at each node. Formulating a day-to-day traffic assignment model as a SRM has two main advantages. First, as the SRM is constructed on a node-link network representation, its problem size is tractable, and the path-non-uniqueness and path-overlapping problems identified in He et al. (2010) can be overcome. Second, various control strategies can be applied on nodes and easily combined into the model, since the flow adjustment is directly formulated at nodes.

However, maintaining the flow conservation is not an easy task in the absence of path flow variables. The LBM in He et al. (2010) essentially embeds a traffic assignment sub-problem to ensure that the link flows satisfy the conservation law. For a large network, an efficient path generation algorithm is needed to solve the traffic assignment sub-problem in the LBM. The SRM in Smith and Mounce (2011) requires an additional computational adjustment of link flows to maintain flow conservation at nodes. This ad-hoc process induces difficulties in analyzing the system properties, especially when the link cost function is non-separable.

This paper proposes a general formulation for link-based deterministic day-to-day traffic assignment models. The proposed generalized flow splitting model not only enables the computation of the link flow trajectories by circumventing the realism issues of path-based models, but also maintains flow conservation endogenously. Since it is developed on the link-node network representation, path variables are completely absent in the formulation, while link flows are updated through a demand distribution matrix to guarantee the flow conservation.

The proposed general formulation can provide modeling flexibility and serve as a unified umbrella for establishing a variety of day-to-day traffic assignment models based on different assumptions on travelers' path swapping decision at nodes. The representation of flow splitting rates can vary to accommodate different routing policies and control strategies in the real world. The realization of the splitting rate may follow either the proportional adjustment used in the SRM, or a projection operator similar to the LBM. Any specified model can directly inherit the analytical properties of the general framework. Thus, the cumbersome analysis due to the exogenous flow adjustment process in the SRM could be avoided.

This remainder of the paper is organized as follows. The next section reviews the SRM proposed in Smith and Mounce (2011) and discusses its inherent problems. Section 3 presents the mathematical formulation of the proposed flow splitting modeling framework, which is established on an acyclic sub-network. The relationship between the modeling framework and existing link-based models is also discussed. Section 4 provides mathematical properties of the proposed flow splitting modeling framework. Section 5 demonstrates the applicability of the proposed modeling framework using a numerical example constructed on a small network. The final section provides some concluding comments.