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Mesoscopic traffic state estimation based on a variational formulation of the LWR model in Lagrangian-space coordinates and Kalman filter

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Abstract

This paper proposes a new model-based traffic state estimation framework using the LWR model formulated in vehicle number – space (Lagrangian – space) coordinates. This formulation inherits the numerical benefits and modelling flexibility from Lagrangian (vehicle number – time) models. Specifically, a variational formulation of the LWR model is selected as the underlying process model. Compared to the traditional conservation law approach in the same coordinate system, the current formulation entitles a simplified expression (no complex state updating originated from different traffic conditions), and provides more accurate numerical results in the prediction step of the data assimilation framework (exact solution to the continuous model when the fundamental diagram is bi-linear). More importantly, this formulation is particularly convenient for data assimilation, because in reality, the flow characteristics are mostly observed at fixed point (spatial fixed) or along vehicle trajectories (vehicle number fixed). These observations are located on cell boundaries of the Lagrangian-space grid, which makes any traffic state estimation method convenient with this approach. Its corresponding observation models are also defined to incorporate both spatial-fixed and moving observations. A Kalman filter framework is applied with the underlying traffic system model. Moreover, travel time can be directly derived from system estimates, and no state transformation is required compared to other estimation approaches. Model validation experiment based on a synthetic traffic network has demonstrated the feasibility of the proposed framework, and suggested promising extensions for future applications.

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Keywords: Traffic state estimation; variational formulation; LWR model; Lagrangian-space coordinates; Kalman filter

1. Introduction

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Traffic state estimation (TSE) and short-term state forecasting are central components in dynamic traffic management and information applications. Generally model-based TSE relies on two components: a model-based component and a data assimilation algorithm. The model-based component consists of two parts: *a*) a dynamic traffic flow model to predict the evolution of the state variables; and *b*) a set of observation equations relating sensor observations to the system state. Thereafter, a data-assimilation technique is adopted to combine the model predictions with the sensor observations. For example, the Kalman filter (KF) (Herrera and Bayen (2010)) and it advanced relatives, such as Extended KF (Wang and Papageorgiou (2005)), Unscented KF (Ngoduy (2008)), Ensemble KF (Work *et al.* (2008)) have been widely applied in the field of traffic state estimation.

The same traffic flow model can be formulated in three two-dimensional coordinate systems regarding space x, time t and vehicle number n. Laval and Leclercq (2013) have presented three equivalent variational formulations of the first-order traffic flow models, namely $N(x,t)$ model, $X(t,n)$ model, $T(n,x)$ model respectively, under the theory of Hamilton-Jacobi partial differential equations. Most of TSE applications are based on the traditional space-time (Eulerian) formulation. Recent studies have shown that a first-order (LWR) traffic flow model can be formulated and solved more efficiently and accurately in vehicle number–time (Lagrangian) coordinates (Leclercq *et al.* (2007)). And its related Lagrangian formulation of state estimation enables more accurate and efficient application of data assimilation methods, due to the solution to the mode-switching problem (traffic information travels in one direction) and less non-linearity of the system model (Yuan *et al.* (2012)).

Furthermore, Yang *et al.* (2015) have investigated the possibility for TSE in the vehicle number-space coordinate systems. Their formulation follows a traditional conservation law approach. The conservation law equation is discretized, and traffic pace is used as the state updating variable. Due to the retainment of spatial coordinates (preserve segment-based representation), traffic information still travels in both directions. This formulation still requires complex state updating matrix for various traffic conditions (upwind and downwind numerical schemes considering both downstream and upstream cells), which is the same as the Eulerian counterpart. The numerical benefit from Lagrangian models is not fully achieved. As argued in Yuan *et al.* (2012), small disturbance for state estimation around capacity point may result in corrections with the "wrong" sign (i.e., the estimator may infer congested traffic while in reality traffic is flowing freely). Alternatively, TSE relied on a variational (Hamilton-Jacobi) formulation of traffic flow models is considered to be much simpler to compute and numerically more accurate under same conditions, compared with the conservation law approach. However, only a few studies have applied such formulations for state estimation purposes. As one of the few examples, Deng *et al.* (2013) extended Newell's three-detector model (the aforementioned N-model, Newell (1993)) for TSE. It provided a more flexible way to assimilate real-world heterogeneous data sources compared to the Eulerian conservation law approach.

This paper proposes a novel mesoscopic model-based traffic state estimation framework using a variational formulation of the LWR model in vehicle number – space (Lagrangian-space) coordinates. This formulation can incorporate the numerical benefits and modelling flexibility of Lagrangian-time models. Specifically, the variational formulation of the LWR model is selected as the underlying process model. Compared to the traditional conservation law approach in the same coordinate system, the current formulation entitles a simplified expression (no complex state updating originated from different traffic conditions), and provides more accurate numerical results in the prediction step of the data assimilation framework (exact solution to the continuous model when the fundamental diagram is bi-linear). Its corresponding observation models are also included to incorporate both spatial-fixed and moving observations. A Kalman filter framework with the underlying model is proposed, and it will be validated on a synthetic network, namely a homogeneous corridor with boundary conditions.

This paper is organized as follows. Section 2 presents the methodology of the proposed TSE framework, including underlying traffic flow model, corresponding observation models and the data assimilation technique. Section 3 illustrates the experimental setup. Loop and probe vehicle data are generated from a synthetic traffic network. Different scenarios are defined to validate the framework under selected performance indicators. Section 4 summarizes the simulation results. Conclusions and recommendations are drawn in Section 5.

2. Methodology

This section defines both the process model and observation models in the state estimation framework. Due to the linear formulation of the traffic system, a linear Kalman filter is used as the data assimilation technique.