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Optimizing emergency transportation through multicommodity quickest paths

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Abstract

In transportation networks with limited capacities and travel times on the arcs, a class of problems attracting a growing scientific interest is represented by the optimal routing and scheduling of given amounts of flow to be transshipped from the origin points to the specific destinations in minimum time. Such problems are of particular concern to emergency transportation where evacuation plans seek to minimize the time evacuees need to clear the affected area and reach the safe zones.

Flows over time approaches are among the most suitable mathematical tools to provide a modelling representation of these problems from a macroscopic point of view. Among them, the Quickest Path Problem (*QPP*), requires an origin-destination flow to be routed on a single path while taking into account inflow limits on the arcs and minimizing the makespan, namely, the time instant when the last unit of flow reaches its destination.

In the context of emergency transport, the *QPP* represents a relevant modelling tool, since its solutions are based on unsplittable dynamic flows that can support the development of evacuation plans which are very easy to be correctly implemented, assigning one single evacuation path to a whole population. This way it is possible to prevent interferences, turbulence, and congestions that may affect the transportation process, worsening the overall clearing time. Nevertheless, the current state-of-the-art presents a lack of studies on multicommodity generalizations of the *QPP*, where network flows refer to various populations, possibly with different origins and destinations.

In this paper we provide a contribution to fill this gap, by considering the Multicommodity Quickest Path Problem (*MCQPP*), where multiple commodities, each with its own origin, destination and demand, must be routed on a capacitated network with travel times on the arcs, while minimizing the overall makespan and allowing the flow associated to each commodity to be routed on a single path. For this optimization problem, we provide the first mathematical formulation in the scientific literature, based on mixed integer programming and encompassing specific features aimed at empowering the suitability of the arising solutions in real emergency transportation plans.

A computational experience performed on a set of benchmark instances is then presented to provide a proof-of-concept for our original model and to evaluate the quality and suitability of the provided solutions together with the required computational effort. Most of the instances are solved at the optimum by a commercial MIP solver, fed with a lower bound deriving from the optimal makespan of a splittable-flow relaxation of the *MCQPP*.

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1. Introduction

In transportation networks with limited capacities and travel times on the arcs, a class of problems attracting a growing scientific interest is represented by the optimal routing and scheduling of given amounts of flow to be transshipped from the origin points to the specific destinations in minimum time. Such problems are of particular concern to emergency transportation where evacuation plans seek to minimize the time evacuees need to clear the affected area and reach the safe zones.

Flows over time approaches are among the most suitable mathematical tools to provide a modelling representation of these transportation problems from a macroscopic point of view. Firstly introduced by Ford and Fulkerson (1958, 1962), these methods rightly capture the dynamic evolution of the scenario e.g. the flow variation over time and the time needed for the transhippment, for a complete survey see Aronson (1989), Koehler et al. (2009). Polynomial-time algorithms have been proposed to solve some of these problems; however, in many general cases their NP-hardness has been proven, see Ford and Fulkerson (1962), Hoppe (1995), Hoppe and Tardos (2000), Hall et al. (2003), Klinz and Woeginger (2004). Flows over time problems may be applied to various real-life situation, e.g. road and air traffic control, communication networks, production systems, and financial flows, see Powell et al. (1995), Aronson (1989).

Among this class of approaches, the Quickest Flow Problem (QFP) asks for sending a s-t flow (namely, a flow directed from a source node s to a sink node t) taking into account the limitations of inflow on the arcs and such that the last unit of flow arrives at destination as quickly as possible, thus minimizing the makespan of the evacuation process. The QFP was shown to be solvable in strongly polynomial time by Burkard et al. (1993). A recent formulation can be found in Lin and Jaillet (2015). The Evacuation Problem is an extension of the QFP to multiple origins and a single sink, see Chalmet et al. (1982), Hamacher and Tjandra (2002). For the multicommodity generalization of the QFP, namely, the Quickest Multicommodity Flow Problem ($MCQFP$), a valid fully polynomial time approximation scheme has been provided in Fleischer and Skutella (2002, 2003) and Hall et al. (2003). Nevertheless, in a real management of emergency transportation operations, a relevant drawback of the solutions provided by the QFP lies in the frequent assignment of multiple and bifurcated paths to the s-t flow demand, requiring thus a group of evacuees to be divided and routed on different ways to destination.

The Quickest Path Problem (QPP), extensively treated in the literature, see Chen and Chin (1990) and Pascoal et al. (2006), copes with this issue requiring the origin-destination flow to be routed on a single path.

State-of-the-art presents a lack of studies on the multicommodity generalization of the QPP, where each commodity, with its own origin, destination, and flow demand, has to be routed on a single path sharing the same capacitated arcs with the other commodities, while minimizing the makespan. We refer to this as the Multicommodity Quickest Path Problem ($MCQPP$), which reveals its dramatic relevance when unsplittable dynamic flows are necessary to prevent all those sources of interference and congestion that may affect the transportation process worsening the overall clearing time.

The literature presents a related problem for static network flows, the Multicommodity Unsplittable Flow problem introduced by Kleinberg (1996) and the Multicommodity k-splittable flow problem, see Baier et al. (2005), Caramia and Sgalambro (2008, 2010), and Gamst (2014). The unsplittable flow request has been studied by Mawson (2005) and Hamacher et al. (2011) for clusters of people evacuating an area in a single-source graph. Their Quickest Cluster Flow Problem is formulated as a non-linear model that accounts for one cluster size and single flow units (clusters with size 1). For each given cluster a unique path to destination is determined. The NP-hardness has been proven for general and tree networks i.e. single source-multiple sinks directed networks and with several sizes for clusters, see Leiner and Ruzika (2011).

In this paper we provide, to the best of our knowledge, the first mathematical formulation in the scientific literature for the $MCQPP$, based on mixed integer linear programming. Our aim is providing a valid mathematical support to the development of evacuation plans to be used in real emergency transportation scenarios. A relevant issue to be considered is therefore the operational implementation of the evacuation plans, that could be affected by human behaviour as a reaction to the emergency situation. To favour an easy compliance of the interested populations to the emergency transport plans, it is important to avoid as much as possible phenomena such as interferences, turbulence, and congestions, arising in some cases for both vehicles and crowd flows. To this aim, we introduce to the above