Available online at www.sciencedirect.com

ScienceDirect

Transportation Research Procedia 7 (2015) 75 – 95

Transportation Research Procedia

www.elsevier.com/locate/procedia

21st International Symposium on Transportation and Traffic Theory

Modeling Absolute and Relative Cost Differences in Stochastic User Equilibrium Problem

Xiangdong Xu[a], Anthony Chen[a,b]*, Songyot Kitthamkesorn[c], Hai Yang[d], Hong K. Lo[d]

[a] *Key Laboratory of Road and Traffic Engineering, Tongji University, Shanghai, China*
[b] *Department of Civil and Environmental Engineering, Utah State University, USA*
[c] *Office of Transport and Traffic Policy and Planning, Bangkok, Thailand*
[d] *Department of Civil and Environmental Engineering, The Hong Kong University of Science and Technology, Hong Kong*

Abstract

This paper aims to develop a hybrid closed-form route choice model and the corresponding stochastic user equilibrium (SUE) to alleviate the drawbacks of both Logit and Weibit models by simultaneously considering absolute cost difference and relative cost difference in travelers' route choice decisions. The model development is based on an observation that the issues of absolute and relative cost differences are analogous to the negative exponential and power impedance functions of the trip distribution gravity model. Some theoretical properties of the hybrid model are also examined, such as the probability relationship among the three models, independence from irrelevant alternatives, and direct and indirect elasticities. To consider the congestion effect, we provide a unified modeling framework to formulate the Logit, Weibit and hybrid SUE models with the same entropy maximization objective but with different total cost constraint specifications representing the modelers' knowledge of the system. With this, there are two ways to interpret the dual variable associated with the cost constraint: shadow price representing the marginal change in the entropy level to a marginal change in the total cost, and dispersion/shape parameter representing the travelers' perceptions of travel costs. To further consider the route overlapping effect, a path-size factor is incorporated into the hybrid SUE model. Numerical examples are also provided to illustrate the capability of the hybrid model in handling both absolute and relative cost differences as well as the route overlapping problem in travelers' route choice decisions.

Selection and peer-review under responsibility of Kobe University

Keywords: stochastic user equilibrium; logit; weibit; absolute cost difference; relative cost difference; route overlapping

* Corresponding author. Tel.: +1-435-797-7109; fax: +1-435-797-1185.
E-mail address: anthony.chen@usu.edu (A. Chen)

2352-1465 Published by Elsevier B.V. This is an open access article under the CC BY-NC-ND license

Selection and peer-review under responsibility of Kobe University
doi:10.1016/j.trpro.2015.06.005

1. Introduction

The user equilibrium (UE) principle proposed by Wardrop (1952) is perhaps the most widely used route choice model in urban and regional transportation planning practices. It assumes that all travelers are rational in terms of preferring the lower travel time routes, have perfect knowledge of network travel times, and are able to identify the minimum travel time route (Sheffi, 1985). These unrealistic assumptions have been widely recognized and criticized by both researchers and practitioners. Among the various relaxations of these assumptions, Daganzo and Sheffi (1977) suggested the stochastic user equilibrium (SUE) principle to relax the perfect knowledge assumption of the UE model. The SUE incorporates a random perception error term in the route cost function to capture travelers' imperfect knowledge of travel times, such that they do not always end up picking the minimum travel time route. Different distribution assumptions on the random perception error term lead to different specialized probabilistic route choice models. A closed-form probability expression and equivalent mathematical programming (MP) formulation are generally preferred in real-world applications. In the literature, there are two main types of closed-form route choice models: Logit (Dial, 1971) and Weibit (Castillo *et al.*, 2008). The Logit and Weibit models were originally derived from the random utility maximization approach by assuming the Gumbel and Weibull random error distributions. Fisk (1980) and Kitthamkesorn and Chen (2013, 2014) provided the equivalent MP formulations for Logit and Weibit models, respectively.

1.1. Motivations and observations

The Logit route choice probability expression is as follows:

$$P_k^w = \frac{\exp\left(-\theta c_k^w\right)}{\sum_{p\in K^w}\exp\left(-\theta c_p^w\right)} = \frac{1}{\sum_{p\in K^w}\exp\left[-\theta\left(c_p^w - c_k^w\right)\right]},\ \forall k\in K^w,\ w\in W, \tag{1}$$

where W is the set of origin-destination (O-D) pairs, K^w is the set of routes between O-D pair w, c_k^w is the travel cost of route k, θ is the dispersion parameter. This model has two known drawbacks: (1) inability to account for overlapping (or correlation) among routes and (2) inability to account for perception variance with respect to trips of different lengths (Sheffi, 1985). These drawbacks stem from the underlying assumptions that the random error terms are *independently and identically distributed* (IID) Gumbel variates with the same and fixed perception variance.

The Weibit route choice probability expression is as follows:

$$P_k^w = \frac{\left(c_k^w\right)^{-\beta}}{\sum_{p\in K^w}\left(c_p^w\right)^{-\beta}} = \frac{1}{\sum_{p\in K^w}\left(c_p^w / c_k^w\right)^{-\beta}},\ \forall k\in K^w,\ w\in W, \tag{2}$$

where β is the shape parameter of Weibull random error distribution. The Weibit model does not require the homogeneous perception variance assumption (i.e., the identically distributed assumption). Therefore, different trip lengths can be identified by the heterogeneous perception variances (Castillo *et al.*, 2008). However, it also has its own limitation of not being able to account for any arbitrary multiplier on the route cost (Kitthamkesorn and Chen, 2014).

Below we use a two-route network to demonstrate the drawbacks of the Logit and Weibit models. We assume that the dispersion parameter (θ) of the Logit model is equal to 0.1, and the shape and location parameters (β and ζ) of the Weibit model are equal to 2.1 and 0, respectively. In Case I and Case II, the upper route cost is larger than the lower route cost by 5 units (i.e., absolute cost difference) for both networks. In the short network, the upper route cost is twice larger than the lower route cost, while it is only less than 5% larger in the long network. In Case III and Case IV, the upper route cost is twice larger than the lower route cost (i.e., relative cost difference) for both networks. In the short network, the upper route cost is larger than the lower route cost by 5 units, while it is 50 units larger in the long network. Herein the relative cost difference is based on the route cost ratio of the Weibit model as shown in Eq. (2), and the absolute cost difference is based on the route cost deviation of the Logit model as shown in Eq. (1).