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On Multi-Objective Stochastic User Equilibrium

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Abstract

There is extensive empirical evidence that travellers consider many 'qualities' (travel time, tolls, reliability, etc.) when choosing between alternative routes. Two main approaches exist to deal with this in network assignment models: Combine all qualities into a single (linear) utility function, or solve a multi-objective problem. The former has the advantages of a unique solution and efficient algorithms; the latter, however, is more general, but leads to many solutions and is difficult to implement in larger systems. In the present paper we present three alternative approaches for combining the principles of multi-objective decision-making with a stochastic user equilibrium model based on random utility theory. The aim is to deduce a tractable, analytic method. The three methods are compared both in terms of their theoretical principles, and in terms of the implied trade-offs, illustrated through simple numerical examples.

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Keywords: network equilibrium; stochastic route choice; multi-objective decision-making; logit model

1. Introduction

It has long been known that there are many qualities, other than travel time, that motivate travellers in their choice of route, such as trip length, tolls and travel time unreliability. For example, from a route choice survey, Abdel-Aty et al. (1995) identified the three most important qualities to be: (1) shorter travel time (ranked as the first reason by 40% of respondents); (2) travel time reliability (32%); and (3) shorter distance (31%). Note that some people chose to indicate more than one quality as most important, which explains the sum being bigger than 100%. In the present paper we are interested in ways in which such multiple qualities may be accounted for in general in a predictive network model, with a specific focus (given its timeliness) on the way in which travellers deal with the potentially competing objectives of choosing a route to minimise their mean travel time and choosing one to minimise travel time unreliability.

Presently there exist two main ways of dealing with multiple qualities in a (deterministic) network user equilibrium (UE) context. The first (single objective) approach is to combine them into a single measure of generalised cost for each route and compute traffic flows that satisfy the Wardrop (1952) user equilibrium condition, which is attained if no user can improve their cost by unilaterally changing their route. A common approach to incorporate several

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route choice qualities is to consider a generalised cost function, which is the sum of monetary cost (such as tolls and vehicle operating costs, which are closely related to distance) and travel time multiplied by a value of time, see e.g. Dial (1979); Leurent (1993); Florian (2006); Chen et al. (2010). Regarding travel time reliability, Lo et al. (2006) formulated a multiple user-class equilibrium model considering travel time and travel time reliability, combined in a single objective as minimising travel time budget, which is defined as the expected travel time plus a travel time margin (or buffer time), with the travel time margin being dependent on the level of risk aversion of each user class. Watling (2006) proposed a late arrival penalised UE (LAPUE) which assumes users minimise a composite path disutility, incorporating the generalised cost plus a late arrival penalty. A few researchers, such as Larsson et al. (2002) have also considered nonlinear generalised cost functions.

The second approach, which has been the subject of more recent research, is to treat the qualities separately and to aim for a multi-objective equilibrium. This approach follows the principle of Pareto optimality or non-dominance commonly applied in multi-objective optimisation: A multi-objective equilibrium is attained if no user can improve any of the route choice qualities without deteriorating at least one other. Wang et al. (2010) showed that this approach is more general than approaches based on (additive) generalised cost functions, even if the latter consider a distribution of the value of time, as proposed by Leurent (1993) or Dial (1996). In fact, there are multi-objective equilibrium solutions that are based on rational route choices, that generalised cost approaches will miss. Wang and Ehrgott (2013) proposed a bi-objective approach considering the qualities travel time and toll, whereas Wang et al. (2014) consider travel time and travel time reliability (measured as standard deviation of travel time) as route choice qualities, and Wang and Ehrgott (2014) propose a multi-objective equilibrium model with travel time, travel time reliability and toll as objectives users aim to minimise.

In Table 1 we summarise other existing approaches from the literature that deal with multiple criteria network user equilibrium models. For each reference, we distinguish between the route choice criteria that have been considered and the path cost objective used in the models. We also state whether the model follows the UE or stochastic user equilibrium (SUE) principle (SO means social optimum) and what source of heterogeneity is considered.

Table 1. Other multiple criteria user equilibrium models.

Reference	Criteria	Objective	UE vs. SUE	Heterogeneity
Jaber and O'Mahoney (2009)	Service charge, time, toll	Generalised cost	SUE	Multiclass VOT, multigroup information
Leurent (1996)	time, cost	Generalised time	UE	VOT distribution
Nagurney (2000)	time, cost	Generalised cost	UE	Multiclass VOT
Nagurney and Dong (2002)	time, cost	Generalised cost	UE	Multiclass VOT
Tzeng and Chen (1993)	time, air polllution, distance	Generalised cost	UE	Discrete set of weights
Yang and Huang (2004)	time, cost	Generalised cost	UE, SO	Multiclass VOT

The single-objective approach has the advantage of typically providing a unique solution, see e.g. Florian and Hearn (1995) and Gabriel and Bernstein (1997), for the case of additive and non-additive path costs, respectively. This is extremely useful for planners when assessing proposed future policies using the network user equilibrium model. Also, efficient computational methods have been proposed for implementing it in large-scale systems (Dial, 2006; Florian et al., 2009; Bar-Gera, 2010; Gentile, 2014). However, the difficulty in specifying or estimating any general form of utility function means that almost always a constant linear form must be assumed, whereas it is not clear that travellers really perceive or trade off qualities in this way. On the other hand, the multi-objective approach has the advantage that it does not need to pre-suppose any relationship between the qualities (it is invariant to a monotone transformation of the qualities). However, its purpose is to generate a whole set of candidate solutions, which is difficult for planners to use in evaluating policy measures, and also gives rise to computational difficulties for identifying such solution sets for anything more than small-scale systems.

In the present paper we aim to take the best elements of each of these approaches. We adopt the basic philosophy of a multi-objective approach, but then aim to derive probability measures which distribute travellers to particular routes, thus aiming for a unique solution. The methods we shall propose extend and/or generalise the well-known single objective stochastic user equilibrium (SUE) model (Daganzo and Sheffi, 1977). In doing so, therefore, they also provide a future pathway to extending efficient algorithms developed for SUE to our new formulations, so that large-scale systems may be solved. The purpose of the present paper is to set out several alternative candidate formu-