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Stochastic Optimal Path Problem with Relays

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Abstract

This paper studies the optimal path problem for travelers driving with vehicles of a limited range, such as most battery electric vehicles currently available in the market. The optimal path in this problem often consists of several relay points, where the vehicles can be refueled to extend its range. We propose a stochastic optimal path problem with relays (SOPPR), which aims at minimizing a general expected cost while maintaining a reasonable arrival probability. To account for uncertainty in the road network, the travel speed on a road segment is treated as a discrete random variable, which determines the total energy required to traverse the segment. SOPPR is formulated in two stages in this paper. In the first stage, an optimal routing problem is solved repeatedly to obtain the expected costs and arrival probabilities from any node to all refueling nodes and the destination. With this information, the second stage constructs an auxiliary network, on which the sequence of refueling decisions can be obtained by solving another optimal path problem. Label-correcting algorithms are developed to solve the routing problems in both stages. Numerical experiments are conducted to compare the stochastic and deterministic models, to examine the impact of different parameters on the routing results, and to evaluate the computational performance of the proposed algorithms.

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Keywords: stochastic optimal path; relays; arrival probability; label-correcting algorithm

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1. Introduction

This paper is concerned with the problem of finding an optimal path between a given pair of nodes for travelers driving with vehicles that have a limited range. If the minimum distance between the node pair exceeds the range limit, the vehicle must be refueled at designated stations. Since a feasible path may thus consist of several relays, the problem is called the optimal path problem with relays (OPPR). OPPR finds applications in crew scheduling, rostering, aircraft routing and telecommunications (Smith et al., 2012). In transportation, OPPR has attracted attention recently thanks to its role in supporting the transition to alternative fuel vehicles (AFV) (e.g. Laporte and Pascoal, 2011; Jiang et al., 2012; Adler et al., 2012). A primary challenge to the adoption of most AFV is that the lack of a dense network of refueling stations causes “range anxiety”, i.e. the drivers’ worries of running out of fuel en route. A long term solution to the problem of range anxiety is to optimize the design of the refueling stations (e.g. Lim and Kuby, 2010; Wang and Wang, 2010; Mak et al., 2013; Nie and Ghamami, 2013; He et al., 2013). In the foreseeable future, however, the drivers of most AFV would still face limited refueling options. In this circumstance, careful routing decisions that integrate travel cost minimization and relay requirements will contribute to the positive user experience (hence the adoption) of AFV.

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The literature on OPPR is relatively sparse. Most existing studies tackle OPPR by taking advantage of the fact that an optimal path must consist of a sequence of optimal subpaths between relay nodes (Lawler, 2001; Cabral, 2005; Adler et al., 2012). Accordingly, this approach divides the original problem into two stages. In the first stage, the optimal paths between each relay node pair are obtained by solving a weight constrained shortest path problem (WCSPP), and then used to create an auxiliary network that contains only the relay nodes. In the second stage, the optimal refueling decision is determined by solving a standard shortest path problem on the auxiliary network. The computational challenge here is to solve WCSPP efficiently, which has been studied extensively in the literature and is known as NP hard (e.g. Gary and Johnson, 1979; Handler and Zang, 1980; Desrosiers et al., 1995; Dumitrescu and Boland, 2003; Chen and Nie, 2013). Adler et al. (2012) consider a special case of OPPR in which the cost to be minimized coincides with the weight constraint, and show that it can be solved in polynomial time. Recently, Laporte and Pascoal (2011) propose a one-stage labeling algorithm for a variant of OPPR where all costs and weights are assumed to be non-negative integers. Their algorithm makes use of the property that the number of labels needed at any node is bounded by the weight limit. Smith et al. (2012) develop both two-stage and one-stage algorithms for OPPR and compare their performance. Their one-stage label-correcting algorithm is built on the preprocessing techniques of Dumitrescu and Boland (2003), and their two-stage algorithm adopts a variation of A* algorithm. According to the numerical experiments, OPPR can be solved efficiently by these algorithms, and their one-stage algorithm is the more efficient of the two in general.

The fuel efficiency, hence the maximum range of a vehicle, depends on the average driving speed (Barth et al., 1996, 2000), which in turn is highly correlated with the prevailing speed of the road on which the vehicle is operated. Consequently, the vehicle's range is also affected by traffic conditions, which vary substantially due to unexpected demand surge and supply disruptions (FHWA, 2006). Optimal path finding under such uncertainty has been well documented, especially along the line of optimizing travel time reliability (Miller-Hooks and Mahmassani, 2000; Waller and Ziliaskopoulos, 2002; Fan et al., 2005; Nie and Wu, 2009; Xing and Zhou, 2011; Wu and Nie, 2011; Huang and Gao, 2012). Yet, uncertainty could have much worse consequences for AFV drivers than longer-than-expected delays. Specifically, a sub-optimal route obtained without considering the impact of uncertainty could substantially increase the risk of failing to reach the next relay point. To the best of our knowledge, few had examined how this risk may be incorporated into finding optimal paths with relays. In He et al. (2014), the range of battery electric vehicles is allowed to vary with traffic conditions. Yet, their work excludes stochasticity, and focuses on deriving user equilibrium conditions for BEV drivers rather than optimal path finding. de Weerdt et al. (2013) consider routing electric vehicles in a time-dependent and stochastic network. While they develop a dynamic programming formulation that aims at maximizing path utility, the risk of failing to reach the destination is not addressed. Fontana (2013) uses a robust optimization approach to solve the optimal route problem for electric vehicles facing uncertainty in traffic conditions. However, they do not consider the recharging of the electric vehicles. Thus, their algorithm may only be used for short trips.

In light of the above gap, this paper proposes a stochastic optimal path problem with relays (SOPPR), which aims at minimizing a general expected cost while maintaining a reasonable probability of arriving at the destination. The problem is formulated in two separate stages. In the first stage, an all-to-one optimal routing problem is solved repeatedly to obtain the expected costs and arrival probabilities associated with non-dominated routing policies from any node to all refueling nodes and the destination. Importantly, the first stage does not yield explicit paths, but rather adaptive routing policies. With this information, the second stage constructs an auxiliary network, for which another all-to-one optimal path problem is solved. The optimal paths resulted from the second stage problem connects the origin and the destination through a sequence of refueling nodes. The complete optimal routing policies can then be retrieved by combining the results from both stages.

The rest of the paper is organized as follows. A range-speed model for electric vehicles is described in Section 2. Section 3 presents the stochastic routing model, followed by the discussion of the solution algorithm given in Section 4. Numerical results are reported in Section 5. Section 6 concludes the paper.

2. The range-speed model

It is well known that the total power required to drive a vehicle can be modeled as a function of driving speed and acceleration (see e.g. the widely used Comprehensive Modal Emissions Model (CMEM) developed by Barth et al.,