Available online at www.sciencedirect.com

ScienceDirect

Transportation Research Procedia 7 (2015) 341 – 361

Transportation Research Procedia

www.elsevier.com/locate/procedia

21st International Symposium on Transportation and Traffic Theory

Optimal Multi-Step Toll Design under General User Heterogeneity

Hongyu Chen [a], Yu (Marco) Nie [a,*], Yafeng Yin [b]

[a] *Department of Civil and Environmental Engineering, Northwestern University, Evanston, IL, 60201 USA*
[b] *Department of Civil and Coastal Engineering, University of Florida, Gainesville, FL 32611*

Abstract

This paper studies the optimal multi-step toll design problem for the bottleneck model with *general* user heterogeneity. The design model is formulated as a mathematical program with equilibrium constraints (MPEC), which is NP-hard due to non-convexity in both the objective function and the feasible set. An analytical method is proposed to solve the MPEC by decomposing it into smaller and easier quadratic programs, each corresponding to a unique departure order of different user classes. The quadratic programs are defined on a polyhedral set, which makes it easier to identify a local optimum. Importantly, each quadratic program is constrained by a set of linear feasibility cuts that define the presence of each user class in the arrival window. We prove that the proposed method ensures global optimality provided that each quadratic program can be solved globally. To obviate enumerating all departure orders, a heuristic method is developed to navigate through the solution space by using the multipliers associated with the feasibility cuts. Numerical experiments are conducted on several small examples to validate the proposed methodology. These experiments show that the proposed heuristic method is effective in finding near-optimal solution within a relatively small number of iterations.

Selection and peer-review under responsibility of Kobe University

Keywords: step toll; general user heterogeneity; mathematical program with equilibrium constraint; quadratic program; bottleneck model

1. Introduction

The peak-time congestion pricing (Vickrey, 1969) has been widely studied in the context of traffic management. Despite its theoretical appeal, peak-time pricing remains unpopular among public. Opponents frequently cite regressive redistributive effects as a main drawback (Small, 1983; Arnott et al., 1992; Evans, 1992). Such an argument hinges on the fact that pricing affects travelers unequally because they value time and schedule punctuality differently. Not surprisedly, the impact of such user heterogeneity on optimal toll design and welfare effects has attracted much attention in the past decades (Small, 1982; Cohen, 1987; Arnott et al., 1992, 1994; Lindsey, 2004; Small et al., 2005; van den Berg and Verhoef, 2011; Liu and Nie, 2011; Hall, 2013). To maximize efficiency, Vickrey's toll has to vary continuously with time. Yet, such a time-varying toll is rarely implemented in practice. Empirical evidence suggests that simpler pricing schemes may have a better chance to rally support. For example, according to a survey conducted in early 1990s (Higgins, 1994), travelers seem to dislike time varying tolls more than flat tolls. Indeed, existing con-

* Corresponding author. Tel.: +1-847-467-0502 ; fax: +1-847-491-4011.
E-mail address: y-nie@northwestern.edu

Selection and peer-review under responsibility of Kobe University
doi:10.1016/j.trpro.2015.06.018

gestion pricing schemes (e.g. London, Singapore and Stockholm, Lindsey et al., 2012) often use "step tolls" that keep toll rates constant in predefined discrete time windows.

This paper is focused on the optimal multi-step toll design problem under *general* user heterogeneity. In our context, user heterogeneity typically consists of the desired arrival time (t^*), the value of time (α) and the values of schedule punctuality (β for early arrival and γ for late arrival). Previous studies often make assumptions about the relationship between α, β and γ to simplify the analysis. The most restrictive assumptions require either that all three parameters vary proportionally (Vickrey, 1973; Xiao et al., 2011), or that only α (Arnott and Kraus, 1995; van den Berg and Verhoef, 2011) or γ (van den Berg, 2014) may vary; hereafter referred to as *proportional heterogeneity*, *α heterogeneity* and *γ heterogeneity*, respectively. These assumptions effectively reduce a three-dimensional problem to a one-dimensional problem. A less restrictive structure of heterogeneity reduces the dimension of the problem to two by allowing α and β to vary freely and assuming γ as a function of β (Cohen, 1987; Newell, 1987; Arnott et al., 1988, 1994; van den Berg and Verhoef, 2011; Liu and Nie, 2011; Hall, 2013).

Welfare effects of congestion pricing seem to depend on the imposed heterogeneity structure. For example, with proportional heterogeneity, all users are better off or break even under Vickrey's time-varying toll (Xiao et al., 2011; van den Berg and Verhoef, 2011), and yet with α heterogeneity, all users are worse off or break even (van den Berg and Verhoef, 2011). Notably, the gains from pricing rise with the value of time in these one-dimensional cases. However, such a monotonic relationship no longer holds with two-dimensional heterogeneity (van den Berg and Verhoef, 2011). Clearly, restricting the heterogeneity structure, which has so far been considered necessary to maintain tractability, also limits the ability to appreciate and draw insights about the complex interactions at work.

Motivated by the above, we propose an analytical method to find optimal step tolls for the bottleneck model with users of general heterogeneity. By general heterogeneity, we mean that no arbitrary relationships are imposed on α, β and γ. [1] The proposed method aims to locate an exact solution by enumerating all possible combinations of user departure orders. The underlying idea behind the method is recently explored in Chen et al. (2015), which focuses on finding user equilibrium solutions for the bottleneck model with general heterogeneity and *given* step tolls. Chen et al. (2015) show that, once the departure order of different user classes is given, a linear equation system can be constructed to generate a candidate solution, which is then verified against the equilibrium conditions. Since the number of possible departure order is finite, the method guarantees finding the correct equilibrium solutions after exhausting all possible departure orders. This paper tackles the optimal toll design problem using the similar idea. Specifically, we construct and solve a toll design problem for each departure order, which is formulated as a quadratic program constrained by user equilibrium conditions associated with the departure order. The optimal step toll configuration can then be determined by comparing the objective functions of all feasible design problems. Note that the general toll design problem constrained by equilibrium conditions is a challenging network design problem known to be NP-hard, even without considering user heterogeneity. Indeed, because the number of possible departure orders grow exponentially with the problem size (the number of users and the number of step tolls), the exact method is impractical except for extremely small problems. In light of this limitation, an effective heuristic method will be developed to obviate the complete enumeration of departure orders.

Few analytical studies [2] had considered a heterogeneity structure as general as pursued in this paper. Lindsey (2004) analyzes a similar model but his focus is to prove that it admits one and only one user equilibrium under mild conditions. His result is significant but does not prescribe solution and design methods. Recently, the user equilibrium problem for the step-tolled bottleneck model under general heterogeneity has been solved using a semi-analytical method (Liu et al., 2015) and an analytical method (Chen et al., 2015). However, neither study addresses the optimal design of step tolls.

The rest of this paper is organized as follows. Section 2 introduces the basic setting of the bottleneck model with general user heterogeneity and step tolls. Section 3 presents the formulations of the optimal step toll design problem as a mathematical program with equilibrium constraints. Development of analytical and heuristic solution methods

[1] We still assume all users share the same desired arrival time t^*. However, different desired arrival times can be easily accommodated using the proposed method, as explained in conclusions (Section 6).

[2] By "analytical" we refer to studies of bottleneck models in which flow and time are not discretized. Examples of considering user heterogeneity in discretized bottleneck models may be found in Yang and Huang (1997); **?**); Ramadurai et al. (2010); **?**.