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On the Existence of Stationary States in General Road Networks

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Abstract

Our daily driving experience and empirical observations suggest that traffic patterns in a road network are relatively stationary during peak periods. In numerous transportation network studies, there has been an implicit conjecture that stationary states exist in a network when origin demands, route choice proportions, and destination supplies are constant. In this study, we first rigorously formulate the conjecture within the framework of a network kinematic wave theory with an invariant junction model. After defining stationary states, we derive a system of algebraic equations in 3-tuples of stationary link flow-rates, demands, and supplies. We then introduce a new definition of junction critical demand levels based on effective demands and supplies. With a map in critical demand levels, we show that its fixed points and, therefore, stationary states exist with the help of Brouwer's fixed point theorem. For two simple road networks, we show that the map is well-defined and can be used to solve stationary states with a brute-force method. Finally we summarize the study and present some future extensions and applications.

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Keywords: Network kinematic wave model; stationary states; demand and supply; critical demand levels; fixed point; existence.

1. Introduction

During peak periods in an urban freeway network, a daily commuter would have almost the same schedule and route everyday and also experience congestion at similar locations and times. Therefore, from the viewpoint of the traffic system, "the traffic demand and origin-destination desires are relatively constant over the time period", and the network reaches a stationary state, in which the locations and sizes of queues are nearly time-independent (Wattleworth, 1967). Such stationary traffic patterns can be observed from the snapshots of speed profiles in the Los Angeles freeway network during the morning peak hours on June 18, 2013, as shown in Figure 1: in the network, congested links, queue lengths, and bottleneck locations remain the same during the peak period from 7:30 to 9:00.

In many studies on analysis, control, management, planning, and design of road networks during peak periods, traffic patterns have been assumed to be stationary (Merchant and Nemhauser, 1978b; Yang and Yagar, 1995; Yang and Lam, 1996): in (Beckmann et al., 1956), the static traffic assignment problem was formulated to determine the aggregate route choice behaviors of vehicles; in (Godfrey, 1969), it was postulated that a network-wide macroscopic fundamental diagram (MFD) exists in such stationary, or steady, states, and this has been verified by observations (Geroliminis and Daganzo, 2008); in (Wattleworth, 1967), the local and global control problem of a freeway system was solved with linear programming methods; in (Potts and Oliver, 1972), network flow conservation problems are solved; and in (Payne and Thompson, 1974), the integrated traffic assignment and ramp metering problem was solved for stationary traffic patterns. Thus, there has been an implicit conjecture that constant demand and route choice

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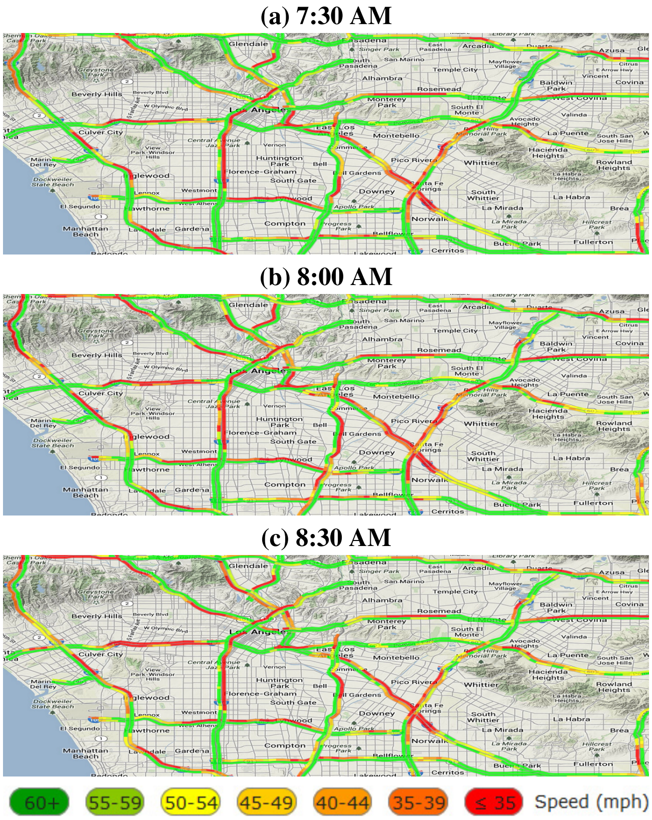

Fig. 1. Stationary traffic patterns in the Los Angeles freeway network during the morning peak period (7:30-8:30) on June 18, 2013 (Data source: `http://pems.dot.ca.gov/`)

patterns lead to stationary patterns in general networks. Even though an understanding of characteristics of stationary states is instrumental for studying various network problems, there has been no theoretical proof or disproof of it.

Furthermore, link performance functions have been widely used to determine travel times from flow-rates on stationary links during peak periods (Beckmann et al., 1956). For example, the BPR link performance functions have been critical for the advancement of transportation network analysis, planning, and design, since they enabled well-defined mathematical programming formulations and numerical solution methods of the static traffic assignment problem in large-scale road networks (Sheffi, 1984; Boyce et al., 2005). However, more and more evidences have shown that link performance functions fail to capture realistic traffic characteristics on links or through junctions in oversaturated networks, as (i) they contradict the fundamental diagram of traffic flow, which suggests that the travel time cannot be uniquely determined by the flow-rate (Greenshields, 1935); (ii) they cannot capture the interactions or competitions among different traffic streams at merging and diverging bottlenecks (Daganzo, 1995a). Such limitations of link performance functions have motivated many studies on dynamic traffic assignment problems (Merchant and Nemhauser, 1978a; Peeta and Ziliaskopoulos, 2001), in which more realistic traffic flow models are used. But such dynamic problems are much more challenging both analytically and computationally. In addition, even though link performance functions are physically limited for congested links, the existence of stationary traffic patterns during peak periods is a reasonable assumption and has been verified by our daily experience and observations. Therefore, a more reasonable next step for traffic assignment is to develop a physically meaningful link performance function for stationary links. Such an undertaking again requires an understanding of characteristics of stationary states in general networks during peak periods.