6th Grade Students’ Solution Strategies on Proportional Reasoning Problems

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Abstract

This research was conducted to investigate 6th grade students’ problem solving strategies and whether these strategies change with problem type and number structure of problems. 165 randomly selected students of grade six participated in this study. A problem test which contains proportional and non-proportional word problems was designed as a data collecting tool. Number structures also considered in the problem test. Descriptive data analysis methods were used in this study. Analysis has shown that students used 7 different strategies on solving proportional problems and 6 different strategies on solving non-proportional problems.

Keywords: Problem Solving Strategies, Proportional Reasoning, Number Structures of Problems

1. Introduction

1.1. Proportional reasoning

Students’ first experiences with mathematics are based on natural numbers in their school life. The first years of primary school includes addition and subtraction that is based on the first-order relationships between countable objects. In the middle school years, students introduce with rational numbers as well as natural numbers. During these years, students must make several major transitions in their mathematical thinking. A central change in thinking is required in a shift from natural number to rational numbers and from additive concepts to multiplicative
concepts (McIntosh, 2013, p. 6). This is an important and difficult conceptual leap for students; mathematical experiences in elementary school focus primarily on countable objects and first-order relationships. In proportional situations students must replace additive reasoning and notions of change in absolute sense with multiplicative reasoning and notions of change in a relative sense (Baxter & Junker, 2001). This second-order relationship is difficult for students because it requires more complicated mental structures than simple multiplication and division. Piaget considered the development of proportional reasoning to be a turning point in the development of higher order reasoning (Aleman, 2007, p. 22). In this sense, the proportional reasoning ability merits whatever time and effort that must be expended to assure its careful development (NCTM, 2000; Ben-Chaim, Fey, Fitzgerald, Benedetto, Miller, 1988; Lesh, Post, Behr, 1988; Lamon, 1993; Baykul, 2009).

Smith (2002) described the importance and complexity of proportionality in this way: “No area of elementary school mathematics is as mathematically rich, cognitively complicated, and difficult to teach as proportionality (Johnson, 2010, p. 3). Many important concepts at the foundation level of elementary mathematics are often linked to proportional reasoning (NCTM, 2000, p. 212). Proportional reasoning is both capstone of elementary arithmetic and the cornerstone of all that is to follow. It therefore occupies a pivotal position in school mathematics programs (Lesh et al., 1988). Using proportional reasoning, students consolidate their knowledge of elementary school mathematics and build a foundation for high school mathematics. Students who fail to develop proportional reasoning are likely to encounter obstacles in understanding higher-level mathematics (Langrall & Swafford, 2000).

1.2. Problem types and number structures

Cramer & Post (1993) categorized proportional tasks as missing-value problems, numerical comparison problems and qualitative prediction and comparison problems. In missing-value problems three pieces of numerical information are given and one piece is unknown. In numerical comparison problems, two complete rates are given. A numerical answer is not required, however the rates are to be compared. Qualitative prediction and comparison problems require comparisons not dependent on specific numerical values. Van Dooren, De Bock, Hessels, Janssens, Verschaffel, (2005) categorized non-proportional tasks (i.e., problems for which a proportional solution was manifestly incorrect but for which another method could be applied to find the correct answer) as additive problems, constant problems and linear problems. In linear problems, the linear function underlying the problem situation is of the form $f(x) = ax + b$ with $b \neq 0$. Additive problems have a constant difference between the two variables, so a correct approach is to add this difference to a third value. Constant problems have no relationship at all between the two variables. The value of the second variable does not change, so the correct answer is mentioned in the word problem.

According to Lesh et al., (1988) proportional reasoning encompasses not only reasoning about the holistic relationship between two rational expressions but wider and more complex spectra of cognitive abilities which includes distinguishing proportional and non-proportional situations. Studies on proportional reasoning has shown that additive strategy is the most frequently used error strategy while students solve proportional problems (Tourniaire, 1986; Karplus, Pulos, Stage, 1983; Bart, Post, Behr, Lesh, 1994; Singh, 2000; Misailidou & Williams, 2003; Duatepe, Akkus, Kayhan, 2005). Similarly, students give proportional responses to non-proportional problems (Duatepe et al., 2005; Van Dooren, De Bock, Vleugels, Verschaffel, 2010; Van Dooren, De Bock, Verschaffel, 2010; De Bock, Van Dooren, Janssens, Verschaffel, 2002; De Bock, De Bolle, Van Dooren, Janssens, Verschaffel, 2003). This shows that students have difficulty in distinguishing proportional and non-proportional problem statements.

Number structure refers to the multiplicative relationships within and between ratios. A ‘within’ relationship is the multiplicative relationship between elements in the same ratio, whereas a ‘between’ relationship is the multiplicative relationship between the corresponding parts of different ratios (Fig. 1) (Steinthorsdottir & Sriraman, 2009).