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On Efficiency Loss of Multiclass Traffic Equilibrium Assignment with Elastic Demand

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Abstract

Seeking the upper bound of the efficiency loss in traffic assignment has attracted many scholars' attentions in the last decade. The existing researches mainly focus on traffic assignment with fixed demand, few on user equilibrium (UE) assignment with elastic demand focus on single user. In this paper, the authors investigate the efficiency loss of multiclass traffic equilibrium assignment (MTEA) with elastic demand. Two decision making criteria are used in this traffic assignment problem, namely, time-based and monetary-based. The variational inequality (VI) models for MTEA under time-based decision criterion and monetary-based decision criterion are formulated. The efficiency loss caused by this two decision criterion is analytically derived and the relations to some network parameters are discussed. It is shown that the upper bound of efficiency loss caused by the time-based assignment depends on the type of link travel time function, the ratio of user benefit and the total social surplus under time-based at traffic equilibrium assignment state. The upper bound of efficiency loss caused by monetary-based assignment depends on the type of link travel time function, the value of time (VOT) of user classes, the ratio of user benefit and the total social surplus under monetary-based at traffic equilibrium assignment state, the ratio of user benefit at system optimum (SO) and the total social surplus at traffic equilibrium state under monetary –based. The numerical tests validate our analytical results.

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Keywords: Multiclass equilibrium traffic assignment; variational inequality; efficiency loss; elastic demand; social surplus

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1. Introduction

The UE and the SO are two famous assignment principles regarding the users' route choice behavior in traffic networks. Since the 1950s, it is well known that the outcome of the user's selfish behavior is generally not identical with the SO. However, the gap between the SO and the UE in the worst case had been unknown for a long time. In the last decade, quantifying the efficiency loss of user's behavior in the transportation network has been a prevalent topic in traffic science. Roughgarden and Tardos (2002) first introduced the efficiency loss or price of anarchy (POA) in traffic assignment and used it to quantify the difference between the UE and the SO, the POA is the worst possible ration between the total cost at UE and the SO. After then, many researchers (Roughgarden and Tardos, 2003; Roughgarden and Tardos, 2004; Correa et al. 2004; Huang et al., 2006; Wu et al., 2008) have extended the above work in different aspects. Recently, researchers investigated the efficiency loss of the equilibrium assignment in the stochastic circumstance (Guo et al., 2010; Yu et al., 2009) and the mixed equilibrium traffic assignment (Liu et al., 2007; Han and Yang, 2009; Guo and Yang, 2009; Yu and Huang, 2010; Karakostas et al., 2011; Liu et al., 2011). However, their researches are only subject to the traffic assignment with fixed demand. Up to now, few studies have been conducted to investigate the efficiency loss of traffic assignment with elastic demand. Chau and Sim (2003) extended Roughgarden and Tardos's results to nonseparable but symmetric cost functions and proposed a bound with symmetric cost function and elastic demand. The POA in elastic demand is defined as the ratio between the maximum social surplus and the total surplus at user equilibrium. Han et al. (2008) derived several bounds for the POA of the noncooperative congestion games with elastic demands and asymmetric linear or nonlinear cost functions. Yang et al. (2010) examined the efficiency loss of a second-best congestion pricing scheme in general networks due to inexact marginal-cost pricing with either fixed or elastic demand. Karakostas and Kolliopoulous (2009) examined how to induce selfish heterogeneous users in a multicommodity network to reach an equilibrium that minimizes the social cost with elastic demand.

This study focuses on the efficiency loss of the MTEA with elastic demand. First, this study formulates the VI models for this multiclass problem under time-based decision criterion and monetary-based decision criterion, respectively. Secondly, it derives the upper bounds of the inefficiency caused by the UE against the SO under the time-based and monetary-based, respectively. Then, the study examines the relationships between the bounds and the network parameters. Finally, it gives a simple numerical example. The value of this study lies in more understanding on complex route choice behavior and the consequences on system performance.

2. MTEA model with elastic demand

A transportation network $G=(N,A)$ composed of a finite set of nodes N, and a finite set of directed links A. Let W be the set of all Origin-Destination (OD) pairs, R be the set of all paths in network and R_w be the set of all paths between OD pair $w\in W$. The travel demand can be subdivided into M classes corresponding to groups of users with different values of time. Let $\beta_m>0$ be the average VOT of user class m. Let q_w^m be the demand of class m between OD pair $w\in W$. Let $B_w^m(q_w^m)$ be the inverse demand function for class m on OD pair $w\in W$. Let v_a^m be the flow of user class m on link $a\in A$ and v_a be the aggregate flow on link $a\in A$. The link travel time function $t_a(v_a)$ is a function of the flow v_a on link $a\in A$ and assumed to be separable, differentiable, convex and monotonically increasing with the aggregate link flow v_a. Let f_{rw}^m be the flow of class m on path $r\in R_w$. Let $c_a^m(v_a)$ be the actual travel cost of class m on link $a\in A$, which is assumed to be a monotonically increasing function of the aggregate link flow v_a. The flow of class m on link a can be expressed in terms of path flows, i.e,

$$v_a^m=\sum_{w\in W}\sum_{r\in R_w} f_{rw}^m\delta_{ar}^w,\forall a\in A,m=1,2,\cdots,M\ . \tag{1}$$

where δ_{ar}^w 1 if path $r\in R_w$ traverses link $a\in A$ and zero otherwise and