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Fluid Approximation of Point-Queue Model

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Abstract

Point-queue model is widely used in the dynamic user equilibrium (DUE) analysis in discrete-time or continuous-time form. In this paper, a continuous time point queue is proposed. In the former studies, the negativity of the queue length of the original point-queue model is shown and some improvement has been made. Based on the observation that the original point-queue model is actually a queuing model with a server and a buffer with infinite capacity, a fluid approximation (FA) model is proposed to interpret the original point-queue model. Three essential components are a flow balance function, an exit flow function and a time-dependent capacity utilization ratio function, which are all in continuous form. During the analysis, the theoretical proof and numerical study of the non-negativity of queue length are accomplished. With the first-order Taylor expansion, this paper applies the Gronwall's inequality to prove the non-negativity of queue-length. Through numerical testing different specific FA models in the solution scheme, we can show that the negativity of the queue length in the FA model is overcome and some differences between the FA and former studies are discussed. Based on the testing, the capability of our model in approximating the point-queue model is demonstrated.

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Keywords: Continuous point-queue model; Fluid Approximation; Gronwall's inequality; Non-negativity; Kendall form

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1. Introduction

During recent decades, dynamic traffic assignment (DTA) has been studied extensively, which can be used in the real-time traffic application, such as variable message signs (VMS), on-line traffic control and so on. Travel choice model and traffic flow component are the two essential parts of DTA and the later one is also called dynamic network loading (DNL) problem.

As a specific problem in DTA research, dynamic user equilibrium (DUE) assumes that some rational behavioral choices, such as to minimize travel time or the generalized costs. Methodologies for DTA/DUE can be generally categorized as simulation-based approach and the analytical approach. For a review, please refer to and Ran and Boyce (1996) and Peeta and Ziliaskopoulos (2001). Ben-Akiva et al. (2001) and Mahmassani (2001) fall into the former one. Among the analytical modeling methodologies used for DUE, including mathematical programming, optimal control and variational inequality (VI) (or equivalently Nonlinear Complementariy Problems (NCPs)) have been considered the most capable for modeling and computing DUE. (Akamatsu, 2001; Friesz et al., 1993; Peeta and Ziliaskopoulos, 2001; Ran and Boyce, 1996; Wie et al., 2002).

In DNL models, the whole link model and the point-queue models are the mostly used ones. The whole link model assumes that the link travel time is a function of link flow (the inflow and the outflow) as $\tau(t)=h(x(t),u(t),v(t))$, where $\tau(t)$ is the link travel time at time t, $x(t)$ is the number of vehicles on the link at time t, $u(t)$ and $v(t)$ are the inflow and outflow of the link at time t. A number of models are the special cases of this model, such as Ran et al., 1993; Ran and Boyce, 1994and 1996; Friesz et al., 1993; Carey and Subrahmanian, 2000; Carey and McCartney, 2002; Carey and Ge, 2007; Xu et al., 1999. Particularly, in Carey and Ge (2007), both the explicit and implicit discretization schemes of the continuous-time whole link model were studied and the explicit scheme may violate FIFO but the implicit scheme will always guarantee FIFO was concluded. Point queue model was first introduced in Vickrey (1969) and the assumption that traffic flow travels in the free flow speed for the entire length of a network link until it gets to the end of the link, where a queue may form is made in the point-queue mode. That is to say, the physical length of the queue is neglected. Therefore, the spillback can never be produced with the point-queue model, contrary to the spatial queue model (Nie et al., 2008). In this paper, the point-queue model is chosen motivated by Ban et al. (2012). Firstly, the discrete-time point-queue models have been studied extensively compared to the continuous-time ones. Secondly, the negativity of the queue length can be derived in the original point-queue model. Finally, the queue forming and discharging process can be captured in the point-queue model, which is more realistic than the whole link model.

A few efforts have been made to refine the negativity of the queue length in the original point-queue model, such as Nie and Zhang, 2005; Ban et al., 2012; Han et al., 2013a,b. Nie and Zhang (2005) have introduced a modification in discrete time to ensure the non-negativity of the queue length. However, the modification is quite *ad hoc*. Ban et al. (2012) have proposed a depth analysis of the original point-queue model and two modified models are presented. One is a Linear Complementarity System (LCS) and the other is an Ordinary Differential Equation (ODE) with a right-hand side that would be Lipschitz continuous if not for the possible discontinuity of the inflow rate function, which is called α -model. Ban et al. (2012) showed the former LCS formulation is connected with the model in Pang et al. (2011) and the model of Nie and Zhang (2005) and the latter one is an approximated ODE formulation of the former LCS formulation whose solutions are shown to converge to a solution of the LCS as the parameter in the ODE goes to infinity. In Han et al. (2013a,b), a key ingredient called a virtual spatial dimension $x\in[0,L]$, where L is the link length, was introduced and with this ingredient, the original point-queue model is described based on the conservation of mass.

Based on the observation that the original point-queue model is actually a queue model with a server and an infinite buffer at the downstream of the link, a fluid approximation (FA) model is proposed in this paper to interpret the Vickrey's model (1969). In the fields of electrical engineering and management science, a fluid approximation model has received increasing attention during the recent decades considering modeling time-dependent queuing systems, while in transportation, the FA is seldom used. In Green and Kolesar (1991), the time was divided into small time intervals and then stationary approximations were used to calculate performance measures based on the service rates and the time-dependent demand rates in each time interval. The remaining queue length from the last interval and the arrival rate for the current interval are considered in the analysis of demand rate. Their study also empirically examined the accuracy of this computationally efficient approximation in multi-server queuing systems