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An inverse problem for the dynamic oligopolistic market equilibrium problem in presence of excesses

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Abstract

The theory of evolutionary variational inequalities fits very well in a lot of optimization problems, like for example the dynamic oligopolistic market equilibrium problem that we describe in this paper. But here, the authors analyze the behavior of control policies whose aim is to regulate the exportation through the adjustments of supply taxes or incentives on the firms. This is considered as a policy-maker optimization problem. This aspect is studied with the help of an inverse evolutionary variational formulation. Moreover, a characterization of the inverse variational inequality with an appropriate evolutionary variational inequality is given. Unlike the paper (Barbagallo & Mauro, 2013), here the authors explore the possibility of presence of production and demand excesses. Moreover, we provide a definition of equilibrium for the firms by using the infinite dimensional duality theory and later we define an optimal regulatory tax. Finally, we present a numerical scheme in order to compute the equilibrium solution of the inverse variational inequality that models the policy-maker's point of view for the dynamic oligopolistic market equilibrium problem in presence of excesses, then we provide a numerical example.

Keywords: Dynamic oligopolistic market equilibrium problem, policy-maker optimization problem, Production and demand excesses, Evolutionary variational inequality, Inverse evolutionary variational inequality, Lagrange multipliers, generalized self-adaptive projection method.

1. Introduction

In this papers the authors want to improve the results obtained in (Barbagallo & Mauro, 2013). More precisely, they give an optimal control perspective on the dynamic oligopolistic market equilibrium problem by allowing the presence of both production and demand excesses. The authors want to describe a policy-maker optimization problem since they permit that control policies may be imposed to regulate the amounts of exportation. In particular, the policy-maker may decide to impose higher taxes or subsidies in order to restrict or encourage the exportation. The starting point is the analysis of the dynamic oligopolistic market equilibrium problem in a supply-demand market between a finite number of spatially separated firms, through an appropriate

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variational inequality (Barbagallo & Cojocaru, 2009b). Here, the equilibrium conditions are given by means of Lagrange multipliers associated with capacity constraints and with production and demand excesses, but these equilibrium conditions can also be given, in a more practical but equivalent way, as a special case of dynamic Cournot-Nash equilibrium principle, see for instance (Barbagallo & Cojocaru, 2009b, Barbagallo & Maugeri, 2011, Barbagallo & Mauro, 2012a, Barbagallo & Mauro, 2012c). Later, the optimal control perspective will be discussed through the help of inverse variational inequalities. Some existence and regularity results for the these inequalities are established. Moreover, the continuity of the equilibrium solution allows us to introduce a numerical method to solve the policy-maker's point of view for the dynamic oligopolistic market equilibrium problem. In particular, after a discretization procedure we solve the finite dimensional problems by using the self-adaptive projection method studied in (He & Liu, 2011).

The equilibrium formulation between the firms involved in this problem, fits in the light of a dynamic noncooperative behavior. Cournot was the author who first analyzed the most trivial but seminal example of such problems considering the noncooperative behavior between two producers of a certain commodity, the so-called duopoly problem (Cournot, 1838). Only with Nash, later, a more complete extension was considered. In particular, Nash introduced m agents in his model nowadays called noncooperative game, each acting according to his own self-interest (Nash, 1950, Nash, 1951).

The problem we consider is evolving in time. The importance of this assumption can be clearly gathered from M.J. Beckmann and J.P. Wallace's sentence in (Beckmann & Wallace, 1969): "The time-dependent formulation of equilibrium problems allows one to explore the dynamics of adjustment processes in which a delay on time response is operating". This is the reason why in (Barbagallo & Cojocaru, 2009b), the authors introduced, for the first time, the dynamic Cournot-Nash equilibrium principle for the dynamic oligopolistic market equilibrium problem. Moreover, the authors studied the related variational formulation in which the time-dependent process was one of the main features and proved the existence and regularity of a dynamic equilibrium solution. There exists a vast literature about existence and regularity results for the solutions to evolutionary variational inequalities, see for instance (Maugeri & Raciti, 2009, Barbagallo, 2007, Barbagallo, 2008, Barbagallo, 2009a, Barbagallo, 2009b, Barbagallo & Cojocaru, 2009a).

It is also remarkable to study the sensitivity of solution, namely to see the consequences on the solution in correspondence of small changes of the data. This analysis is present in (Barbagallo & Maugeri, 2011) where the authors show that, under suitable assumptions, small changes of the solution happen in correspondence with small changes of the profit function. Moreover, in (Barbagallo & Maugeri, 2011) the authors, for the first time, describe the behavior of the market by means of the Lagrange multipliers taking into account the infinite dimensional duality theory developed in (Maugeri & Raciti, 2010, Daniele & Giuffrè, 2007, Daniele, Giuffrè & Maugeri, 2007, Daniele & Giuffrè, 2009). Such results make use of a particular algebraic definition of interior of sets, called quasi-relative interior (Borwein & Lewis, 1992), that allows to overcome the difficulty of the emptyness of the topological interior of the ordering cone which defines the constraints of many equilibrium problems. It is worth to emphasize that the main aim of Lagrange multipliers is to highlight the presence of constraints in the model so that it is easier to analyze the behavior of the market.

In (Barbagallo & Maugeri, 2010) and (Barbagallo & Di Vincenzo, 2011), the adjustment processes in an oligopolistic market equilibrium model have been represented by means of a memory term which depends on previous equilibrium solutions according to the Volterra operator. In this way, it is possible to take into account of the delay on time response, that is a consequence of the finite speed of propagation of information through the economic network.

The model presented in (Barbagallo & Cojocaru, 2009b) has been improved in a more realistic way by introducing the production excesses in (Barbagallo & Mauro, 2012a) and both production and demand excesses in (Barbagallo & Mauro, 2012c). In order to clarify the introduction of the model with both production and demand excesses we consider some concrete economic situations. During an economic crisis period the presence of production excesses can be due to a demand decrease in demand markets and, on the other hand, the presence of demand excesses may occur when the supply can not satisfy the demand especially for fundamental goods. Moreover, since the market model evolves in time, the presence of both production and demand excesses is a consequence of the fact that the physical transportation of commodity between a firm and a demand market is evidently limited, therefore, there can exist some time intervals in which some of the demand markets require more commodity, though some firms produce more commodity than they can send to all the demand markets. Finally, it is worth to remind that the existence and regularity results, proved in (Barbagallo & Mauro, 2012a,