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A study on delay-sensitive cellular automata

Souvik Roy

Department of Information Technology, Indian Institute of Engineering Science and Technology, Shibpur, Howrah, West Bengal, 711103, India

HIGHLIGHTS

- Cellular automata with delay and probabilistic loss of information is introduced.
- The effect of proposed CA in the dynamics of ECA and Game of Life is presented.
- Phase transition for both ECA and Game of Life is found.

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ABSTRACT

Classically, in cellular automata, no delay in information sharing among the neighbouring cells is considered. However, this assumption is not generally true for natural complex systems (such as physical, biological, social systems) and distributed systems where information sharing (non-uniform) delay cannot generally be ignored. Moreover, sometimes in complex and distributed systems, messages are lost, which makes the system non-deterministic. In this context, the effect of (non-uniform) delay and probabilistic loss of information during information sharing between neighbours in the dynamics of elementary cellular automata and Game of Life is presented in this study. We study the wide variety of results, which include the study of *phase transitions*, for both the elementary cellular automata and Game of Life using a statistical experimental approach.

1. Introduction

A cellular automaton (CA) is a class of dynamic systems for modelling systems with local interactions. A CA is defined over a regular grid, each cell of which consists of a finite automaton that interacts with its neighbours to go to its next state [1]. The beauty of CA comes from simple local interaction which produces a (global) complex behaviour. Since their inception, cellular automata (CAs) have been utilized as an effective tool to model many natural complex (such as physical, biological, social, etc.) systems, distributed systems and networked systems, etc. [2,3]. CAs have always been a natural choice for modelling complex and distributed systems because of their built-in parallelism. Classically, in any cellular automaton, if a cell wants to get state information about its neighbours, the cell gets the information instantly in the next time step. That is, no delay in information sharing among the neighbouring cells is considered. In real natural complex, distributed and networked systems, on the other hand, information sharing delay cannot generally be ignored [4–6]. Most importantly, the delay of information sharing with neighbour cannot be uniform in any natural complex and distributed system.

In this context, this paper develops a different type of cellular automata where a cell shares its state with its neighbours with some delay. The delays of different neighbouring cells may be different. Information sharing in the proposed system is non-deterministic in nature, which implies that a cell shares its state information with its neighbours probabilistically.

E-mail address: svkr89@gmail.com.

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1.1. Notes on the history of delay (in CA)

In the literature of cellular automata, few researchers have explored a new type of asynchronism where a cell shares its state information with its neighbours probabilistically [7,8]. If a cell does not share its state with a neighbour, then it implies that the delay of information passing is infinite. Therefore, the works of Ref. [7,8] have dealt in some sense with delay during information passing. However, if a cell agrees to share its state with any one of its neighbours, it can do it within one time step only. This implies that either there is no delay during state sharing, or there is uniform delay, which is less than the duration of one time step. This fact can be seen in the models of Ref [7,8].

Different kinds of cellular automata models with time delay (for modelling purpose, i.e. cancer tumours, SEIR (susceptible–exposed–infected–recovered) epidemic, etc.) also have been proposed by CA researchers [5,6]. Iarosz et al. defined a cellular automata model for cancer tumours which included the time delay between the time in which the site is affected and the time in which its variable is updated, see Ref. [5]. This work have analysed the effect of time delay on the dynamics through cluster counting. However, the work have dealt with non-uniform time delay in general, but the time delay for all the cells is constant (i.e. uniform) for a particular time step. Recently, Sharma and Gupta [6] have presented a two-dimensional cellular automata model to simulate *SEIR* epidemic spread with time delay. The work of [6] have shown that the delay plays a central role to control disease progression in an infected host. In this context also note that Alonso-Sanz [9,10] have investigated the effect of memory of delay type in the dynamics of elementary cellular automata. Here, the 'memory of delay type' indicates that every cell retains historic memory of its past states. However, this study develops a different type of cellular automata where a cell shares its state with its neighbours with some delay. The works of [11–13] ([11] for one-dimensional case and [12,13] for Game of Life) have also alternatively dealt with 'probabilistic loss of information' in terms of *coupling parameter* (κ) which locally smooths the rigid specification of deterministic cellular automata rules and makes them fuzzier.

1.2. Overview of the proposed model

In this scenario, we target to study a cellular automaton system where delay is not uniform. Let us consider, a cell is updated at time t and cell has two neighbours (left and right). In traditional CA, the two neighbours both can see the updated state information in next time step. In the proposed model, we question this assumption. We consider here that at time $t+1$, both the neighbours may not be aware of the cell's update; only one (or none) can see the update at time $t+1$, and other can observe the update at time $t' > t+1$. This consideration makes the proposed CA more suitable for modelling natural complex and distributed systems.

Now, we develop our proposed CA system which involves delay during information sharing between two neighbours. It only concentrates on the delay when a cell shares its state information with its neighbours. In the proposed system we introduce a non-negative integer function $D(i, j)$ for each i and j where $i \neq j$ and i and j are two neighbouring cells of CA. $D(i, j)$ defines the delay involved in sharing of information of cell i with cell j. If $D(i, j) = 0$, then any update of cell i is immediately seen by cell j. If $D(i, j) = 1$, then the update of cell i is seen by cell j within 1 time step. This implies that if cell i updates its state at time t, then that updated state information is available to cell j at time $t+1$. In traditional cellular automata where delay is not explicitly mentioned, either $D(i, j) = 0$ or $D(i, j) = 1$ for each pair of i and j where cell i and cell j are neighbours to each other. In the proposed system, $D(i, j)$ can be any non-negative integer value. However, following property is obeyed by D in our system.

- $D(i, j) = D(j, i)$ for any pair of neighbouring cells i and j.

Apart from this, the following property is also desirable for our system.

- If i and j are neighbours, then $D(i, j) \geqslant 1$.

The above properties of D (that is, delay) are also suitable for any natural complex and distributed systems. Here, the delays are non-uniform in space, i.e. $D(i, i')$ may be different from $D(j, j')$, where i and i'; and j and j' are neighbouring cells. Note that the delays are uniform in time. However, the delays are deterministic here. Sometimes in natural complex and distributed systems messages are lost, which makes the system non-deterministic. So apart from 'delay', we introduce a probabilistic loss of information during state information sharing in our model.

The 'delay', in our system is not part of the cell's local rule, and the cells are governed by the local rule. When a cell acts to update its state, the cell has to know the neighbours' states. In our system, each cell has a *view* about the states of its neighbours. The view may change from time to time depending on the arrival of state information about neighbours. However, the cells act depending on the current state information about neighbours at that time. Therefore, each cell has to maintain the state information of neighbours to get a view of neighbours' states. When we simulate this system, we use some memory for each cell to store the state information of neighbours. The memory is updated if a new state information is arrived.

As an example, let us consider, a simple 3-cell, 3-neighbour cellular automaton of Fig. 1(a). State of the cells are given within the circles. Consider $D(0, 1) = D(1, 2) = 1$ and $D(0, 2) = 2$. Let us consider that the cells act at time t. Now, each cell has to know the states of its neighbours. Fig. 1(a) shows that each cell has a *view* about the states of neighbours. Cell 0 *sees* that state of cell 1 is 1 (noted over the right side link) and state of cell 2 is 0 (noted over the left link). Now, assume