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Practical boundary element method for piled rafts

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ABSTRACT

In this paper, a practical boundary element method (BEM) is developed to analyze piled rafts. The raft is modeled using the direct boundary element formulation for thick plates; which is suitable for such type of problems. An innovative methodology to add stiffness matrices to the plate integral equations is developed. Hence, pile–soil–raft interactions are added as a special case of the developed methodology. In order to avoid generating huge systems of coupled equations, an effective condensation technique of piles and soil degrees of freedom (DOFs) at pile–soil–raft interface is employed. Several numerical examples are presented and results are compared to previously published results. Then, a practical application is solved to demonstrate the strength and versatility of the proposed formulation.

1. Introduction

Piled rafts are an important type of foundations for tall buildings. This type is suitable to overcome both bearing capacity and settlement problems. A typical review of such importance is presented in Refs. [1–4]. Therefore, accurate modeling of piled raft components: plate, soil, and piles is essential in foundation structural analysis.

Plates rested on soil could be represented as plates over Winkler springs or over elastic half space (EHS). The Winkler model represents the soil as individual springs as in the work of Coduto [5]. Analysis of plates over elastic foundations (Winkler or EHS) were considered using the finite element method as in the work of Cheung and Zienkiewicz [6], Svec and McNeice [7], Svec and Gladwel [8] for thin plates, and Rajapaks and Selvadurai [9] for thick plates. Examples of using the boundary element method for such a problem are the work of Katsikadelis and Armenakas [10], Syngellakis and Bai [11], Paiva and Butterfield [12] for thin plates, and Rashed and co-workers [13–16] for thick plates.

Alternatively, the soil could be modeled based on 3D finite elements or boundary elements. There are two methods to represent the soil using BEM. The first, as 3D boundary elements; which need enclosing boundaries as those of FEM. Such a technique has never been reported in the literature. The second way is to represent the 3D soil medium as elastic half space. It has be noted, that the second way was referred to, by almost all authors in the literature, as “using the boundary element method”, regardless of which numerical model of raft plate was used (either it was modeled using the FEM or the BEM).

Modeling piled rafts is more sophisticated than modeling plates on elastic foundations. Piles should be modeled as embedded in the soil continuum considering all interaction between piles, soil, and raft. The finite element method could be used to represent piled raft, in which three-dimensional solid elements [17] are used to model soil. It has to be noted that, such a model generates a large number of DOFs; therefore, it is inconvenient for practical applications. Alternatively, two-dimensional axisymmetric finite element [18] were used to model axisymmetric foundations, in which interaction effects were ignored. Such a model is also not practical, as it cannot model general practical foundations.

Paiva and co-workers [19,20] developed a BEM formulation for solving piled rafts, in which all the interactions were considered. In [19,20] the pile cap is divided into triangular thin finite elements, despite the conclusion of Small [21], which indicates that the use of thin plate theory may lead to inaccuracy in modeling thick foundations. All piles and soil in [19,20] were embedded in the problem integral equation; therefore, such formulation leads to a huge system of equations leading to difficulty in solving practical piled rafts.

Engineers and modelers, in design offices, tend to use approximate methods for piled raft analysis. They model such a foundation as plate resting on springs. The initial pile stiffness is computed based on empirical formulas according to design code equations [22] or obtained from pile–load test [23].

Few software packages that consider the effect of pile–pile interactions in piled raft analysis are available. ELPLA [24] represents the raft

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as thin plate via plate bending finite elements and considers interactions between piles/soil based on Mindlin's solution (The Elastic approach). LUSAS [25] uses thin/thick plate bending elements to simulate rafts. The soil is represented as 3D finite elements. Piles are modeled as beam elements inside the 3D soil continuum. PLAXIS [26], ABAQUS [27], and Midas [28] are alternative multi-purpose and general FEM based software packages. They could be used to analyze a piled raft system, in which the soil is modeled as 3D finite elements. Despite the spread of such approaches, they are not suitable for large practical applications.

It has to be noted that in all previously considered publications, all verification examples and solved problems (which in some publications referred to as "practical applications") are simple problems and cannot be regarded as practical applications.

In conclusion, in order to develop a practical numerical analysis for piled rafts, the following points should be considered:

(1) Suitable representation of raft as shear-deformable plate to account for the raft thickness.
(2) Suitable modeling of stress concentration due to presence of both pile reactions and applied column loads.
(3) Suitable numerical method to model large practical problems with large number of DOFs.
(4) Suitable implementation of all interaction effects.

This paper considers the boundary element method (BEM) for shear-deformable plates to model thick piled rafts. The use of BEM ensures the continuity of internal variables with no physical discretization as those in the FEM. This ensures the efficient and accurate modeling of singular internal field even when concentrated loads are presented, such as applied column loads or supporting pile reactions. Moreover, piles DOFs are condensed at the head of piles and added as additional stiffness to the integral equation of the plate. This developed technique reduces the number of DOFs considerably to allow solving large-scale problems as presented in Section 6. All interaction effects are considered. Pile–pile interactions is considered based on both the elastic approach [1,3] and the load transfer approach [2]. Pile–soil interactions is also considered based on Mindlin's solution [29]. Soil–soil interactions can be considered as well based on Mindlin's [29], Boussinesq's [30], and Steinbrenner's [31] solutions.

2. Boundary integral equations for plates in bending

In this section, the direct boundary element formulation for thick plates is reviewed. Consider a general thick plate of domain Ω and boundary Γ as shown in Fig. 1. The indicial notation is used, where the Greek indices vary from 1 to 2 and Roman indices vary from 1 to 3. The relevant integral equation could be written as follows:

$$C_{ij}(\xi)u_j(\xi)+\int_{\Gamma(x)} T_{ij}(\xi,x)u_j(x)\mathrm{d}\Gamma(x)=\int_{\Gamma(x)} U_{ij}(\xi,x)t_j(x)\mathrm{d}\Gamma(x)+L_1+L_2 \tag{1}$$

where $T_{ij}(\xi, x)$, $U_{ij}(\xi, x)$ are the two-point fundamental solution kernels for tractions and displacements respectively [15]. The two points ξ and x are the source and the field points respectively. $u_j(x)$ and $t_j(x)$ denote the boundary generalized displacements and tractions. $C_{ij}(\xi)$ is the free term. The integrals L_1, L_2 are the prescribed domain load integrals and could be defined as follows [32]:

$$L_1=\int_{\Gamma(x)}\left[V_{i,n}(\xi,x)-\frac{\nu}{(1-\nu)\lambda^2}U_{i\alpha}(\xi,x)n_\alpha(x)\right]q(x)\mathrm{d}\Gamma(x) \tag{2}$$

$$L_2=\sum_{N_{cell}}\int_{\Omega(L)}\left[U_{ik}(\xi,L)-\frac{\nu}{(1-\nu)\lambda^2}U_{i\alpha,\alpha}(\xi,L)\delta_{3k}\right]P_k(L)\mathrm{d}\Omega(L) \tag{3}$$

where $V_i(\xi, x)$ is a suitable particular solutions to account for uniform domain loading [15], other type of loading could be dealt with via cell loadings. The symbols ν and λ denote the plate Poisson's ratio and shear factor, respectively. $q(x)$ denotes the intensity of the uniform domain loading. N_{cell} denotes the number of loading cells that having domain denoted by Ω_L, P_k denotes the three prescribed loads (two moments and one force) for each loading cell. The field point L denotes the loading cell center.

After discretizing the boundary of the plate into N_{ele} quadratic elements, each node has three unknowns. Eq. (1) could be re-written in a matrix form as follows:

$$\underset{3N\times 3N}{[H]}\times\underset{3N\times 1}{\{u\}}=\underset{3N\times 9N_{ele}}{[G]}\times\underset{9N_{ele}\times 1}{\{t\}}+\underset{3N\times 1}{\{LV\}} \tag{4}$$

where N is the number of nodes. $[H]$ and $[G]$ are the well-known influence matrices. The vector $\{LV\}$ contains prescribed domain loading and loading cells effects (L_1+L_2). After re-ordering the matrices in Eq. (4) to decouple the prescribed values from the unknown boundary values, Eq. (4) could be re-written as follows:

$$\underset{3N\times 3N}{[A]}\times\underset{3N\times 1}{\{u\}\ \text{or}\ \{t\}}=\underset{3N\times 1}{\{LV\}-\{PBV\}} \tag{5}$$

where the vector {PBV} contains prescribed boundary integral values and defined as follows:

$$\{PBV\}=\begin{cases}-\displaystyle\int_\Gamma T^*_{ij}(\xi,x)\mathrm{d}\Gamma(x),\ \text{if } u(x)\text{ is prescribed}\\ -\displaystyle\int_\Gamma U^*_{ij}(\xi,x)\mathrm{d}\Gamma(x),\ \text{if } t(x)\text{ is prescribed}\end{cases} \tag{6}$$

3. The proposed integral equation for plates with additional stiffness

In the previous section, the direct boundary element formulation for a thick plate without any domain supporting substructures is reviewed. In this section, such a plate is reconsidered when additional domain supporting substructures (Ω_s) are present. If the plate in Fig. 1 is supported over (n) substructures (noting that each substructure is divided into a series of supporting cells), Eq. (1) could be re-written as follows:

$$C_{ij}(\xi)u_j(\xi)+\int_{\Gamma(x)} T_{ij}(\xi,x)u_j(x)\mathrm{d}\Gamma(x)$$
$$=\int_{\Gamma(x)} U_{ij}(\xi,x)t_j(x)\mathrm{d}\Gamma(x)+L_1+L_2+\sum_{s=1}^{s=n}I_s \tag{7}$$

In which,

$$I_s=\sum_{N_{c_\Omega}}\int_{\Omega(s)}\left[U_{ik}(\xi,y_s)-\frac{\nu}{(1-\nu)\lambda^2}U_{i\alpha,\alpha}(\xi,y_S)\delta_{3k}\right]F_k(y_s)\mathrm{d}\Omega_s(y_s) \tag{8}$$

where (N_{c_Ω}) denotes the number of the overall supporting cells. $F_k(y_s)$ denotes three unknown generalized forces for each supporting cell. The field point (y_s) denotes supporting cell center. Considering the discretized form of the problem, Eq. (7) could be re-written as follows (after