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Trans-accuracy elements and their application in BEM analysis of structurally multi-scale problems

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ABSTRACT

When solving structurally multi-scale problems with small or slender components using BEM, different sized boundary elements are inevitably required to simulate all kinds of related geometries. In this paper, a family of trans-accuracy boundary elements are constructed based on Lagrange interpolation formulation. These elements not only can fulfil the transition between small and large elements, but also can simulate a whole or a part of end surfaces of cylinders accurately using very few nodes. Based on the constructed trans-accuracy elements, a BEM analysis method of structurally multi-scale problems (MSBEM) is proposed. Three numerical examples for media including different numbers of slender components such as fibers are given to validate the correctness and demonstrate the potential of the proposed methods.

1. Introduction

Recent years, numerical analyses of structurally multi-scale engineering problems have received considerable attentions from researchers. The frequently used method is the finite element method (FEM) [1,2]. However, the boundary element method (BEM) [3,4] has distinct advantages over FEM, attributed to the use of the singular fundamental solutions. The feature of the fundamental solutions rapidly decaying with distance makes the established boundary integral equation very sensitive to the distance and thus can guarantee the final established algebraic equations linearly independent even for very closed collocation nodes as occurred when treating the small-sized or slender components inlayed in a large computational region.

In the analysis of the structurally multi-scale problems using BEM, different sized boundary elements are inevitably required to simulate all kinds of related geometries, that is, small sized elements are used to discretize small or slender components and large elements are used to discretize the matrix medium. To guarantee the computational accuracy and using as few nodes as possible, a kind of trans-accuracy elements are needed to fulfil the transition from small to large sized boundary elements.

In functionality, the frequently used serendipity elements [5] in FEM are kind of the trans-accuracy elements, which are able to fulfil the transition between different orders of finite elements. The serendipity elements are among the most popular element types in the FEM analysis [6]. They provide a finite element subspace which has significantly smaller dimension than the Lagrange element family. However, usually only the lowest degree serendipity elements are discussed, with the pattern for higher degrees not being evident.

In this paper, the idea of using the serendipity element in FEM is borrowed to analyze the structurally multi-scale thermal and mechanical problems using BEM. A family of trans-accuracy boundary elements are constructed based on Lagrange interpolation formulation. These elements not only can fulfil the transition between small and large elements, but also can simulate a whole or a part of end surfaces of cylinders accurately using very few nodes. Based on the constructed trans-accuracy elements, a BEM analysis method of treating multi-scale problems (MSBEM) is proposed in the paper. Several numerical examples for media including different numbers of slender components such as fiber bundles are given to validate the correctness and demonstrate the potential of the proposed methods.

2. Boundary element formulation based on semi-closure elements

For simplicity, the potential problems are discussed in the follow, but the results can also be applied to the elasticity problems.

2.1. Boundary element formulation for potential problems

In three-dimensional potential problems, the boundary integral equation can be expressed as [3]

$$c(P)u(P) + \int_{\Gamma} q^*(Q,P)u(Q)d\Gamma(Q) = \int_{\Gamma} u^*(Q,P)q(Q)d\Gamma(Q) \tag{1}$$

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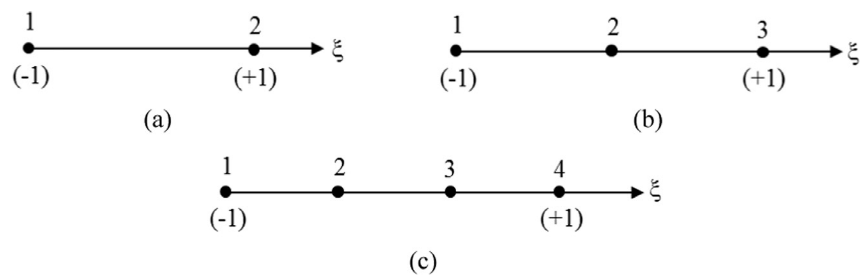

Fig. 1. Frequently used line elements: (a) 2-node linear; (b) 3-node quadratic; (c) 4-node cubic.

where $c = 1/2$ for smooth boundary points and $c=1$ for interior points; P and Q represent the source and field points, respectively; u and q are the potential and flux defined on the boundary Γ; $u^*$and $q^*$are the fundamental solutions for potential problems, which can be expressed as [7,8]

$$u^* = \frac{1}{4\pi r} \tag{2a}$$

$$q^* = \frac{\partial u^*}{\partial n} = \frac{-1}{4\pi r^2}\frac{\partial r}{\partial n} \tag{2b}$$

in which, r is the distance between the source point P and the field point Q; n is the unit outward normal to the boundary Γ.

To evaluate the boundary integrals included in Eq. (1), the boundary Γ of the problem is discretized into a series of boundary elements. The potential and flux over each element can be expressed in terms of their nodal values through shape functions:

$$\begin{aligned} u &= \sum_{k=1}^{m} N_k u^k \\ q &= \sum_{k=1}^{m} N_k q^k \end{aligned} \tag{3}$$

where m is the number of element nodes, N_k is the shape function for the k-th node over the boundary element, and u^k and q^k are the values of the potential and flux at node k, respectively. Eq. (3) is suitable for smooth boundary nodes. For boundary corner nodes, the flux may be discontinuous. In this case, the discontinuous elements are used to deal with the corner nodes [7,16].

Assuming that the boundary Γ of the problem is discretized into n_e boundary elements and using Eq. (3), the discretized form of Eq. (1) can be written as

$$\begin{aligned} cu(P) = &\sum_{e=1}^{n_e}\left\{\sum_{k=1}^{m} q^k \int_{\Gamma_e} u^*(Q,P)N_k(Q)d\Gamma(Q)\right\} \\ &- \sum_{e=1}^{n_e}\left\{\sum_{k=1}^{m} u^k \int_{\Gamma_e} q^*(Q,P)N_k(Q)d\Gamma(Q)\right\} \end{aligned} \tag{4}$$

It is noted that for elasticity problems, a similar equation to Eq. (4) can also be written. The details can be found in related references [9,10].

2.2. Boundary elements for geometry and field variable interpolations

The boundary elements used for geometry computation and field variables (u and q) interpolation in Eq. (3) can be classified into four groups as presented below.

2.2.1. Line elements for 2D boundary

Fig. 1 shows three types of frequently used line elements over the boundary of a two-dimensional problem [3].

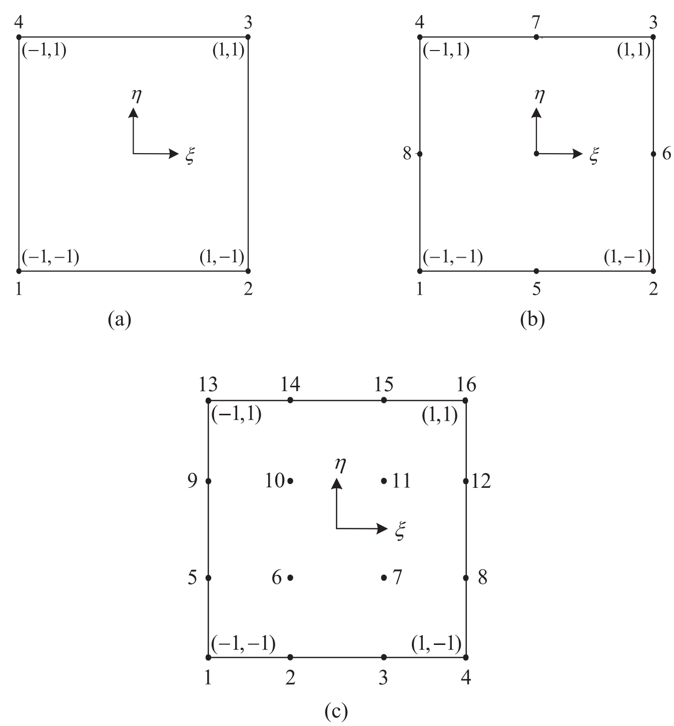

Fig. 2. Frequently used surface elements: (a) 4 node linear; (b) 8 node quadratic; (c) 16 node cubic.

2.2.2. Surface elements for 3D boundary

In order to solve structurally multi-scale problems using BEM, different sized boundary elements are inevitably required to simulate all kinds of related geometries, such as large elements which are used to discretize the matrix medium. Fig. 2 shows three frequently used 4 node linear, 8 node quadratic, and 16 node cubic elements over the boundary of a three-dimensional problem [11].

2.2.3. Semi-closure elements for slender body surfaces

To simulate a structurally multi-scale problem with slender bodies as components using BEM, the semi-closure elements for slender body surfaces are required, such as tube and disk elements constructed based on the Lagrange polynomial interpolation formulation [12–14]. The tube elements are used to discretize the side surface of a tube, a bar or a cylinder, and the disk elements are served to seal the two end surfaces of these components.

To construct a tube element or disk element, the hole element [15] is constructed first. Fig. 3 shows the 4-node, 6-node and 8-node hole elements, the shape functions of which can be found in Appendix A. The details for how to construct these hole elements can be seen in Ref. [13].

Tube elements:

Fig. 4 shows the nodal distributions over three types of the tube elements.