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Minimum mass laminate design for uncertain in-plane loading

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ABSTRACT

Requirements for lower emissions and operating costs make mass reduction of composite structures a significant issue for future aircraft. Here, minimisation of normalised elastic energy under an uncertain, general in-plane loading is used to indicate laminate efficiency and by equivalence minimum mass. Results are the first to investigate the comparative robustness of standard and non-standard angles to uncertain loading. They indicate that weight reductions of up to 8% can be achieved if optimum design, using standard angle ($\theta = 0^{\circ}, \pm 45^{\circ}$ or 90°) and industrial design rules, is replaced by optimising non-standard angles ($0^{\circ} \leq \theta \leq 180^{\circ}$) directly for uncertain loading. However, greater reductions of up to 20% are possible through alignment of laminate balancing axes with principal loading axes. As such, a non-standard angle design strategy is only shown to be warranted if the demonstrated non-uniqueness of optimum designs can be exploited to improve other performance drivers.

1. Introduction

Minimum mass aerospace laminate design is a multi-constraint problem. All relevant failure modes such as buckling, damage tolerance, bolt bearing and notched strength should be considered in order to produce a minimum mass design that delivers the required performance. However, such a complex approach is not justified in the initial design stage. Netting analysis, which ignores the support of the resin matrix and aligns fibres in principal directions to carry principal stresses, leads to laminate designs in which the stresses in fibres are limited to some value associated with failure i.e. fully-stressed fibre design. Verchery [1] has shown that Netting analysis, can be treated as a limiting case of Classical Laminate Theory. His approach indicates that designs with fewer than three fibre directions produce mechanisms when subject to small disturbances in loading. This reveals the reasoning behind established aerospace laminate design practice of using four standard angles (SAs) (0°, $+45^{\circ}$, -45° and 90°) and a design rule of a 10% minimum ply percentage to provide a level of redundancy against loading uncertainty [2]. In contrast, non-standard angle (NSA) designs permit the use of all possible fibre angles ($0^{\circ} \leq \theta \leq 180^{\circ}$) providing greater scope for stiffness tailoring. The advantages of tailoring have been demonstrated through use of lamination parameters and NSA layups over quasi-isotropic layups in optimisation procedures of wing structure solutions for aero-elastic tailoring purposes [3,4], for increased panel buckling performance [5,6], as well as enabling certain types of stiffness couplings [7]. NSAs have also been extensively studied for their use in winding angles for optimising pressure vessel strength [8,9]. However, a lack of specific design rules for NSA laminates can lead to optimum aerostructure designs for specific loadings that, in the extreme Netting analysis regime context, form mechanisms with any perturbation in load. Such laminates rely on the weak resin matrix to prevent collapse under a varying load state. In this paper, to avoid problems of robustness, both NSA and SA laminates are designed considering an uncertain in-plane loading with the use of anti-optimisation, allowing design for all loading scenarios that could be applied. This ensures the structure is designed for the worst case loading i.e. the critical condition limiting the mass of the structure under consideration. Anti-optimisation describes the min-max or max-min optimisation technique whereby a design is optimised to have the best possible worst case performance for the range of uncertainties considered [10] e.g. maximising the minimum buckling load from the range of loads that could be applied from a defined uncertainty in loading, as is the case in Adali et al. [11]. The authors found that, a deterministic design is seen to underperform in buckling performance compared to a robust design when uncertain loads are applied. Anti-optimisation is usually a two-step optimisation process with one optimisation nested within the other [10]. However, here the worst case performance for a range of loadings is found analytically, similar work is also shown by Adali for buckling design under uncertain in-plane loads [12]. Composite laminate uncertainties are also associated with the material and the manufacturing process [12,13] but this not considered in this paper. Optimising for a loading uncertainty has the potential to replace the requirement for a

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10% minimum ply percentage rule in SA designs and allows the use of NSAs without an equivalent constraint. An equivalent NSA 10% rule has been applied by Abdalla et al. [14] using the in-plane lamination parameter space to allow a selection of designs which have a base stiffness in all directions. Similarly a technique ensuring a minimum degree of isotropy in NSA laminate optimisations is shown by Peeters and Abdalla [15] ensuring robust laminates. However, instead of an identical robustness in all directions, the robustness of a design may be better tailored if the range of loads that could be applied are known.

In order to compare design approaches that use SAs and NSAs, laminate in-plane elastic energy under combined bi-axial and shear loading is used to assess laminate efficiency in this paper. Elastic energy minimisation or compliance energy minimisation is a computationally efficient technique that uses either topology or orientation of materials with directional properties, to produce the structures with maximum efficiency. Structures with optimum efficiency take advantage of directional material stiffness properties to produce a minimum global strain state. This requires the structure to have the greatest global stiffness for a given volume of material. Prager and Taylor [16] first outlined optimality criteria justifying the technique of minimisation of elastic energy (subject to given loads) to produce a structure with optimal efficiency. Pedersen [17] subsequently applied this technique to composite materials to find analytical solutions for orientation of a single ply angle subject to in-plane loading. Solutions for multi-layered anisotropic laminates are provided for multi-axial design loadings. Minimisation of in-plane elastic energy in laminate design does not directly imply maximisation of in-plane strength of a composite material. Nevertheless, it is assumed to be sufficient to capture the in-plane strength relationship as fibres are aligned to best carry the applied multi-axial stresses, which is the case for maximum in-plane strength design in a Netting analysis regime [2,18]. Thus the performance of a laminate under a vector of loading can be shown by the single attribute of in-plane elastic energy. In the following sections, laminates are first optimised using the techniques presented with a Genetic Algorithm before designs are analysed and the data presented in plots revealing the potential benefits and drawbacks of new and current methodologies.

2. Minimum mass laminate design

In this section, a process is defined that minimises in-plane elastic energy under fixed and uncertain in-plane multi-axial loadings (axial, transverse, shear) in order to find distributions of SA and NSA plies that maximise laminate efficiency and thus minimise mass. Design constraints for both SA and NSA laminates, in the form of stacking sequence rules, are also derived.

2.1. In-plane elastic energy

Given that the in-plane Hookean or elastic energy for a linear elastic solid is

$$u = \frac{1}{2}\int \boldsymbol{\sigma}^T \boldsymbol{\varepsilon} dV \tag{1}$$

Considering the in-plane laminate stiffness matrix, $\bar{\boldsymbol{Q}}$, the strain terms in Eq. (1), $\boldsymbol{\varepsilon}$, can be substituted for $\bar{\boldsymbol{Q}}^{-1}\,\boldsymbol{\sigma}$, allowing the elastic energy to be expressed using solely laminate level stresses, $\boldsymbol{\sigma}$. These stresses (load per unit laminate cross-sectional area) can be further substituted with the equivalent in-plane loads per unit width, $\boldsymbol{N}$, divided by the laminate thickness, T.

$$u = \frac{1}{2}\int \frac{\boldsymbol{N}^T}{T}\bar{\boldsymbol{Q}}^{-1}\frac{\boldsymbol{N}}{T} dV = \frac{1}{2}\int \boldsymbol{\sigma}^T\bar{\boldsymbol{Q}}^{-1}\boldsymbol{\sigma} dV \tag{2}$$

Assuming a balanced laminate ($\bar{Q}_{16} = \bar{Q}_{26} = 0$) and working per unit volume (allowing for laminate geometry to be ignored) further implies

$$U = \frac{1}{2}\boldsymbol{\sigma}^T\bar{\boldsymbol{Q}}^{-1}\boldsymbol{\sigma} = \frac{1}{2}(q_{11}\sigma_x^2 + 2q_{12}\sigma_x\sigma_y + q_{22}\sigma_y^2 + q_{66}\tau_{xy}^2) \tag{3}$$

where q terms are from the inverse of the laminate stiffness matrix. Division of Eq. (3) by the sum of the squares of the principal stresses normalises U, removing the effect of the magnitudes of the loads/stresses, and allows for an equal comparison between loading states of the same magnitude i.e.

$$\bar{U} = \frac{q_{11}\sigma_x^2 + 2q_{12}\sigma_x\sigma_y + q_{22}\sigma_y^2 + q_{66}\tau_{xy}^2}{2(\sigma_I^2 + \sigma_{II}^2)} \tag{4}$$

where

$$\sigma_{I,II} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \tag{5}$$

The misalignment angle, η, of the principal loading from the balancing axes (about which $+\theta$ and $-\theta$ plies are evenly distributed to prevent extension-shear coupling) is shown in Fig. 1 and defined as

$$\eta = \frac{1}{2}\tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) \tag{6}$$

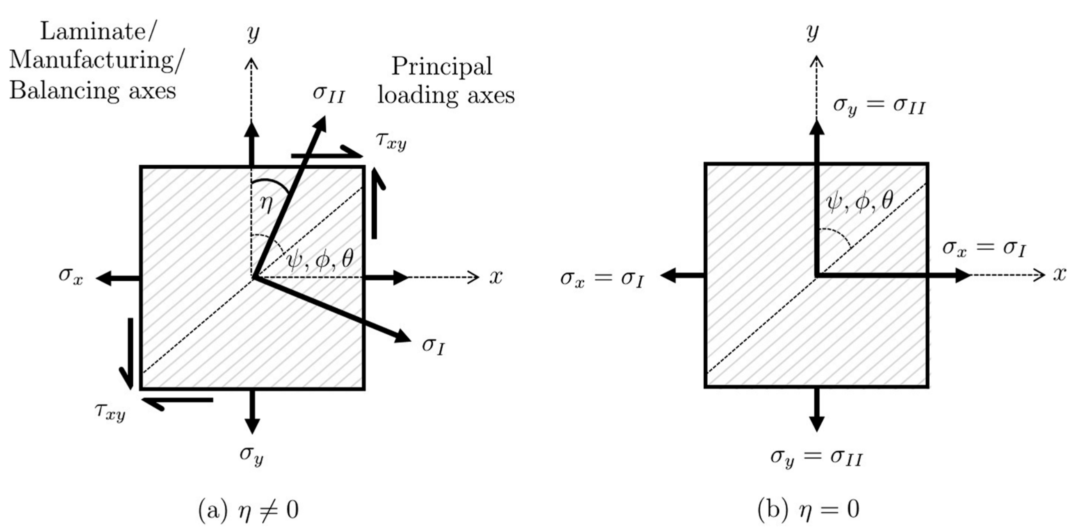

Fig. 1. (a) Diagram showing laminate (x,y) axes (from which ply angles (ψ ,ϕ, θ) are defined and balanced) and principal loading axes offset from balancing axes by angle η. For (b) $\eta = 0$ and thus the balancing axes are aligned with the principal loading axes.