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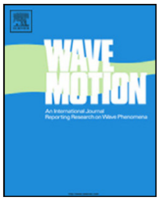

Generation of mode-2 internal waves in a two-dimensional stratification by a mode-1 internal wave

Jianjun Liang [a,b,*], Tao Du [c], Xiaoming Li [a,d], Mingxia He [b]

[a] *Key Laboratory of Digital Earth Science, Institute of Remote Sensing and Digital Earth, Chinese Academy of Sciences, Beijing, China*
[b] *Ocean Remote Sensing Institute, Ocean University of China, Qingdao, China*
[c] *College of Oceanic and Atmospheric Sciences, Ocean University of China, Qingdao, China*
[d] *Key Laboratory of Earth Observation, Hainan Province, Sanya, China*

HIGHLIGHTS

- A theoretical generation model for mode-2 IWs is derived.
- The model captures the generation of mode-2 IWs by an evolutionary mode-1 IW.
- The favorable environmental conditions for the generation is verified.

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ABSTRACT

The generation of mode-2 nonlinear internal waves (IWs) by the evolution of a mode-1 IW in a two-dimensional stratification is investigated. A generation model accounting for intermodal interaction is derived based on a multi-modal approach in a weakly nonlinear and non-hydrostatic configuration. The generation model is numerically solved to simulate the evolution of mode-1 and mode-2 IWs in an inhomogeneous pycnocline. The numerical experiments confirm that mode-2 IWs are generated due to linear and nonlinear intermodal interaction. The mode-2 IW continues growing and gradually separates with the mode-1 IW during the generation process. The numerical results suggest that the pycnocline strength or thickness prominently affects the generation of mode-2 IWs, followed by pycnocline depth. A weakening or thinning pycnocline favors the generation of mode-2 IWs by evidently enhancing linear and nonlinear intermodal interaction, whereas a shoaling pycnocline favors a rapid growth rate mainly by enhancing linear intermodal interaction. The wave amplitude of an initial mode-1 IW strongly affects the generation of mode-2 IWs and increasing it can noticeably enlarge mode-2 IWs.

1. Introduction

Nonlinear internal waves (IWs) are a common phenomenon in the coastal oceans and marginal seas [1,2]. Previous studies show that they have profound impacts on a variety of issues, such as offshore drilling operations [3], underwater acoustic propagation [4] and sediment resuspension [5]. In theory, IWs can be described in terms of vertical mode [6]. The first mode (mode-1) IWs have an in-phase displacement of isopycnals in the vertical direction whereas the second mode (mode-2) IWs have an out-phase behavior. Like mode-1 IWs, mode-2 IWs also take two types of waveforms: convex and concave [7]. For

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[*] Correspondence to: Institute of Remote Sensing and Digital Earth, Chinese Academy of Sciences, No.9, DengZhuang Nan Road, Haidian, 100094, Beijing, China.

E-mail address: liangjj@radi.ac.cn (J. Liang).

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mode-2 convex IWs, the upper isopycnals are displaced upward while the lower isopycnals are displaced downward such that a bulge forms in the middle of a water column. Mode-2 IWs mentioned in the following content are all in the convex type. To date, mode-1 IWs have been extensively studied, covering from generation to dissipation, whereas mode-2 IWs have received less attention. Despite of efforts from laboratory experiments, numerical simulations, *in-situ* measurements and satellite observations having been made to investigate the generation of mode-2 IWs, our understanding of their generation within the ocean interior is still incomplete.

The earliest laboratory experiment of mode-2 IWs comes from the work of Davis and Acrivos [8]. They noticed that mode-2 IWs were easily generated by creating a disturbance to a thin density gradient layer, a transition layer between two deep homogeneous layers. Later, Kao and Pao [9] excited a mode-2 IW by inducing the collapse of a mixed region in a thermocline region; Mehta et al. [10] excited a mode-2 IW by allowing the whole head of a gravity current to intrude into a three-layer fluid with a sufficiently wide middle layer. With a focus on a slope-shelf topography, Helfrich and Melville [11] found that the breaking instability of a mode-1 IW near the shelf break led to the generation of a mode-2 IW. Inspired by the local generation mechanism of IWs in the central Bay of Biscay proposed by New and Pingree [12,13], Mercier et al. [14] reproduced the process and detected the response of mode-2 IWs.

Numerical experiments have revealed various generation mechanisms for mode-2 IWs. These mechanisms include: (i) a steady flow passing over isolated topography when resonant generation occurs [15]; (ii) a mode-1 IW interacting with a steep sill [16] or shoaling over shelf-slope topography [17,18]; (iii) impingement of an internal tidal beam on a pycnocline from below when the horizontal phase speed of the tidal beam matches the eigen-speed of mode-2 IWs [19]; (iv) nonlinear disintegration of mode-2 internal tides [20]; (v) polarity conversion of a concave mode-2 IW [21,22]. Moreover, recent simulation incorporating interaction of barotropic tides with a subcritical ridge shows that the third mechanism works effectively when both the tidal Froude number and contribution to an internal tidal beam from mode-2 waves are high enough [23].

In-situ measurements of mode-2 IWs have been reported in the Middle Atlantic Bight [5], on the shelf of the northern South China Sea [7,24,25], on the northern Heng-Chun Ridge south of Taiwan [26], on the New Jersey Shelf [27], and on the Mascarene Plateau [28,29]. Differing from the former generation mechanisms, Ramp et al. [26] proposed a lee wave mechanism when a tidal current flows over a ridge and Liu et al. [25] suggested a mode-1 IW disintegration mechanism when the mode-1 IW evolves in a horizontally and vertically varying stratification.

Satellite observations of mode-2 IWs are limited, e.g., reported in da Silva et al. [20], Liu et al. [25] and Dong et al. [30]. The study by Dong et al. [30] suggests a different generation mechanism that mode-2 IWs could be induced by an anticyclonic eddy.

Among the various generation mechanisms for mode-2 IWs, the present study aims to clarify the generation of mode-2 IWs by an evolutionary mode-1 IW in a two-dimensional stratification and find the environmental conditions that favor the formation of mode-2 IWs. The two-dimensional stratification is specified by an inhomogeneous pycnocline with a flat bottom, such as a case reported in an in-situ observation [31]. With the goal of quantifying the examined situation as illustrated in Fig. 1, a generation model that accounts for intermodal interaction is derived based on a multi-modal approach. The multi-modal approach has been successfully used in multiple studies of internal waves, such as development of internal solitary waves in various thermocline regimes [32], internal tide generation at the continental shelf [33] and multi-modal evolution of wind-generated long internal waves in a closed basin [34]. All of the previous works are restricted to a one-dimensional stratification, that is, density is only varying in the vertical direction. Here, the multi-modal approach is extended to a two-dimensional stratification, that is, density is varying in both horizontal and vertical directions.

The paper is organized as follows. A theoretical generation model for mode-2 IWs is derived by a multi-modal approach in Section 2. In Section 3, various numerical experiments are set up based on the theoretical generation model. Numerical results are discussed in Section 4 and conclusions are summarized in Section 5.

2. Derivation of the generation model

In this section, we examine the generation of mode-2 IWs by a weakly nonlinear non-hydrostatic mode-1 IW evolving in a two-dimensional ambient density field. A geostrophic current,$(0, V, 0)$, emerges within the region with horizontal density gradient and vanishes outside the region. The system of equations governing the wave motion consists of the continuity equation, the Euler equations and the incompressibility condition. They take the form in a two-dimensional configuration with the Boussinesq approximation:

$$\begin{aligned}
&u_x + w_z = 0,\\
&u_t + uu_x + wu_z - fv = -p_x,\\
&v_t + uv_x + wv_z + uV_x + wV_z + fu = 0,\\
&w_t + uw_x + ww_z = -p_z - \sigma,\\
&\sigma_t + u\sigma_x + w\sigma_z = N^2 w + M^2 u.
\end{aligned} \tag{1}$$

Here, subscripts denote partial derivatives; the surface of the fluid is located at $z = 0$ and z is positive upward; u, v and w are the longitudinal, transverse and vertical velocity components induced by waves, respectively; t is the time; f is the constant Coriolis parameter; p is the density normalized pressure perturbation; σ is the density perturbation with respect to its local static value (multiplied by g/ρ_0; g the gravitational acceleration and ρ_0 a constant reference density); $N^2 = -g\overline{\rho}_z/\rho_0$