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Structural balance in signed networks: Separating the probability to interact from the tendency to fight

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ABSTRACT

Structural balance theory implies hypothetical network effects such as “the enemy of an enemy is a friend” or “the friend of an enemy is an enemy.” To statistically test such hypotheses researchers often estimate whether, for instance, actors have an increased probability to collaborate with the enemies of their enemies and/or a decreased probability to fight the enemies of their enemies. Empirically it turns out that the support for balance theory from these tests is mixed at best. We argue that such results are not necessarily a contradiction to balance theory but that they could also be explained by other network effects that influence the probability to interact at all. We propose new and better interpretable models to assess structural balance in signed networks and illustrate their usefulness with networks of international alliances and conflicts. With the new operationalization the support for balance theory in international relations networks is much stronger than suggested by previous work.

1. Introduction

Generalizing Heider’s theory of cognitive balance (Heider, 1946), Cartwright and Harary (1956) called a signed network *structurally balanced* if every cycle has an even number of negative ties. They proved that a network is balanced if and only if its actors can be divided into two groups with only positive ties within the groups and only negative ties between groups. The fundamental claim of structural balance theory is that actors have a preference for balanced states and if a network is unbalanced then actors have a tendency to increase balance by adapting their ties.

Structural balance has been analyzed in different areas ranging from international relations (Crescenzi, 2007; Maoz et al., 2007; Lerner et al., 2013; Doreian and Mrvar, 2015), interpersonal affective networks (Yap and Harrigan, 2015), political discourse (de Nooy and Kleinnijenhuis, 2013; Kleinnijenhuis and de Nooy, 2013) opinion networks (Altafini, 2012), over to Web-based interaction (Leskovec et al., 2010; Lerner et al., 2012).

Probabilistic rules derived from structural balance theory claim that the probability of positive or negative interaction depends on the signs of indirect ties via third actors. For instance, in the context of international relations, Maoz et al. hypothesized that pairs of countries having a common enemy[1] are more likely to become allies and less likely to fight each other (Maoz et al., 2007).

The central claim of our paper is that empirically estimating the *marginal* probability of positive or negative ties as a function of signed indirect ties has unclear implications for the validity of balance theory. Instead we propose that estimating the *conditional* probability of a tie having a particular sign, given that there is a tie, is a more appropriate operationalization and has a clearer interpretation. More generally, we argue that a separation of the probability of signed ties into the probability to interact at all and the conditional probability to interact negatively can be very insightful—also if the focus of the analysis is on other network effects than structural balance.

Before elaborating our central claim in detail and providing empirical evidence for it, we will sketch our reasoning in the following. For sake of clarity we focus at the moment on a single hypothesis namely that “actors that share a common enemy have a lower probability to be enemies themselves, compared to a random dyad.” Note that this is one of the hypotheses formulated by Maoz et al. (2007, p. 102). At first sight this hypothesis seems to follow quite naturally from structural balance theory. Indeed, if actors *A* and *B* have a common enemy *C* and if a negative tie between

[1] For ease of readability we say that actors connected by positive ties are “friends” and actors connected by negative ties are “enemies” regardless of the actual meaning of signed ties.

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A and B was created, then the triad A-B-C would become unbalanced which—according to balance theory—actors try to avoid. Thus, seemingly, the probability of a negative tie among enemies of enemies should be lower than the baseline probability of negative ties among all dyads. However, the latter conclusion ignores that other network effects could influence A and B to interact more with each other; after all, (A, B) is not a random dyad but one characterized by having a common enemy. By chance alone, a higher interaction probability could also increase the probability of negative interaction between A and B, even in situations in which enemies of enemies interact rather friendly than hostile if they interact. Indeed, in Section 3 we show that in networks of international cooperation and conflict, countries having common enemies have a higher marginal probability to fight each other, compared to a random pair of countries. (Note that this result has also been made by Maoz et al. (2007); we repeat and extend their analysis later in our paper.)

We argue that such an empirical finding is not evidence against *balance theory* but rather it is evidence against a certain *method to test* balance theory. In the same empirical data about international relations we show that, conditional on the presence of interaction, countries that share a common enemy are more reluctant to fight each other (and more likely to become allies) compared to a random pair of countries that do interact. The latter finding also turns out to be very robust with respect to the inclusion or exclusion of variables that control for other network effects.

The key to design more appropriate statistical tests for structural balance theory lies in carefully distinguishing between the tendency to interact at all and the preference (or reluctance) to fight rather than collaborate if interaction takes place. A general decomposition that achieves this distinction is elaborated in Section 2. Empirical evidence is provided in Section 3 where we test balance theory in international relations networks alternatively with models that estimate the marginal probability of positive or negative ties and with models that condition on the presence of interaction. Section 4 discusses further implications of our findings, including how the conditional analysis could be done in more sophisticated network models such as exponential random graph models and stochastic actor-oriented models. The next section reviews previous work on structural balance that is related to our contribution.

1.1. Related work

Heider (1946) postulated that if a person P has a positive attitude towards another person O, then P's attitude towards an entity X should coincide (both positive or both negative) with P's perception of O's attitude on X. In contrast, if P is negatively linked to O then the P-X dyad should have the opposite sign of the O-X dyad. The fundamental claim of balance theory is that actors have a preference for such balanced structures and that they tend to remove imbalances by adapting their ties. Cartwright and Harary (1956) generalized Heider's theory to larger and not necessarily complete signed networks, i.e., networks of n actors in which pairs are either connected by a positive or a negative tie or are not tied at all. They called a signed network balanced if every cycle has an even number of negative ties and proved that a network is balanced if and only if its actors can be divided into two groups with only positive ties within groups and only negative inter-group ties.

Note that Heider's distinction between attitudes and actors' perceptions on the attitudes of other actors gets ignored in the definition of Cartwright and Harary which has been criticized among others in Doreian (2004). While we agree that this distinction is crucial in general, it is of less importance in our paper: our concern with certain methods to statistically assess structural balance applies independently of whether we analyze attitudes or perceptions of attitudes.

Empirical support for structural balance theory has been mixed and findings about imbalances in social networks often lead to refining, augmenting, or generalizing structural balance theory or it lead researchers to explain observed patterns with alternative theories. For instance, Davis (1967) proposes a generalized version of structural balance in which, among others, the triad with three negative ties is not considered as imbalanced. Doreian and Mrvar (2009, 2014) see some violations of structural balance as resulting from other processes such as mediation, differential popularity, and internal subgroup hostility. Maoz et al. (2007) argue that realist theories of political behavior can explain imbalances in networks of international relations. Leskovec et al. (2010) showed that some behavioral patterns in online interaction that are inconsistent with structural balance can be well explained by status theory. Doreian and Krackhardt (2001) found that structural balance theory is supported if the *p*-*o* dyad is positive but contradicted if it is negative. Furthermore, they found that the number of signed triplets increases over time whenever *pq* and *oq* have the same sign, independent of the sign of *po*.[2] Thus, also in their work, the popularity of *q* seems to matter more than the balance of the *p*-*o*-*q* triad. In contrast to the last-mentioned papers, our work here does not seek to assess the validity of balance theory per se but makes a methodological contribution to do so. While, obviously, structural balance has to be confronted with other network theories, we emphasize that empirical tests to assess balance theory against competing theories crucially rely on valid statistical methods.

An insight related to the methodological contribution of our paper is given by de Nooy who analyzed structural balance in networks of positive or negative reviews among literary authors and critics. As de Nooy writes

> "*In my case, the presence or absence of a line (literary evaluation) is not the important phenomenon to be explained because it depends on events and constraints outside the power of the actors in the network.*" [. . .] "*As we will see, it is possible and interesting to predict the sign of an evaluation, conditional on the presence of an evaluation, from the pattern of signs of previous evaluations.*" (de Nooy, 2008, Introduction, Paragraphs 5 and 7)

While we also recommend to analyze the *conditional* sign of ties, our reasoning is different: we claim that even in situations in which the presence or absence of a tie can be explained by factors endogenous to the network, it might still confound balance effects. We argue and demonstrate that structural balance theory reliably explains the sign of a tie, conditional on the presence of a tie; on the other hand, empirical support for balance effects on the marginal probability of positive or negative interaction is weak. Besides de Nooy (2008), other papers that analyze the *conditional* probability of positive or negative interaction include Brandes et al. (2009), Lerner et al. (2012, 2013), de Nooy and Kleinnijenhuis (2013). Note however that these papers, including de Nooy (2008), do not compare their results with the analysis of the *marginal* probability of signed ties and therefore do not clarify whether this could lead to different results.

General model frameworks that can deal with complex statistical dependence in network data include exponential random graph models (ERGMs) for cross-sectional data (e.g., Robins et al., 2007; Lusher et al., 2013), temporal ERGMs (e.g., Hanneke et al., 2010; Cranmer and Desmarais, 2011; Krivitsky and Handcock, 2014), and stochastic actor-oriented models (SAOMs) (e.g., Snijders, 2005). While these model families have been mostly applied to binary network data (where ties can be either present or absent but have no sign) they have recently been generalized to signed networks,

[2] Note that Doreian and Krackhardt analyzed person-to-person networks and replaced Heider's symbol for the entity X with the symbol *q* denoting a third person.