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Agent based models and opinion dynamics as Markov chains

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ABSTRACT

This paper introduces a Markov chain approach that allows a rigorous analysis of agent based opinion dynamics as well as other related agent based models (ABM). By viewing the ABM dynamics as a micro-description of the process, we show how the corresponding macro-description is obtained by a projection construction. Then, well known conditions for lumpability make it possible to establish the cases where the macro model is still Markov. In this case we obtain a complete picture of the dynamics including the transient stage, the most interesting phase in applications. For such a purpose a crucial role is played by the type of probability distribution used to implement the stochastic part of the model which defines the updating rule and governs the dynamics. In addition, we show how restrictions in communication leading to the co-existence of different opinions correspond to the emergence of new absorbing states. We describe our analysis in detail with some specific models of opinion dynamics. Generalizations concerning different opinion representations as well as opinion models with other interaction mechanisms are also discussed. With their obvious limitations, the models do not allow for a direct generalization to more realistic cases, their treatment is only the first step in the stochastic analysis of the micro–macro link in social simulation. We find that our method may be an attractive alternative to mean-field approaches and that this approach provides new perspectives on the modeling of opinion exchange dynamics, and more generally of other ABM.

1. Introduction

Recent improvements in multidisciplinary methods and, particularly, the availability of powerful computational tools are giving researchers an ever-greater opportunity to investigate societies in their complex nature. The adoption of a complex systems approach allows the modeling of macro-sociological or economic structures from a bottom-up perspective – understood as resulting from the repeated local interaction of socio-economic agents – without disregarding the consequences of the structures themselves on individual behavior, emergence of interaction patterns and social welfare.

Agent based models (ABM) are at the leading edge of this endeavor. When designing an agent model, one is inevitably faced with the problem of finding an acceptable compromise between realism and simplicity. If many aspects are included into the agent description, the model might be plausible with regard to the individual behaviors, but it will be impossible to derive rigorous analytical results. In fact, it can even be very hard to perform systematic computations to understand the model dynamics if many parameters and rules are included. On the other hand, models that allow for an analytical treatment often oversimplify the problem at hand. In ABM, we can find the whole spectrum between these two extremes. While simplicity is often favored by physicists in order to be able to apply their well-developed tools from statistical physics, more realistic descriptions are often desired by researchers in the humanities because they are interested in incorporating into the model a reasonable part of their qualitative knowledge at the micro and macro scales. Both views have, of course, their own merits.

Our paper is a contribution to interweaving two lines of research that have developed in almost separate ways: the Markov chain approach and ABMs. The former represents the simplest form of a stochastic process while the latter puts a strong emphasis on heterogeneity and social interactions. The main expected output of our Markov chain strategy applied to ABM is a better understanding of the relationship between microscopic and macroscopic dynamical properties. Moreover, we aim to contribute not only to the understanding of the asymptotic properties of ABM but also to the transient mechanisms that rule the system on intermediate

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time scales. For practical purposes this is the most relevant information for two reasons: first, in our case the chains are absorbing, so the asymptotic dynamics is trivial and second, they describe the evolution of the system before external perturbations take place and possibly throw it into a new setting.

Agent-based opinion models, a particular case of ABM, are among the most simple models in the literature and are therefore a suitable starting point for the analysis. Especially for *binary opinion models* several results have been obtained by previous authors using analytical tools, as shown in the review on social dynamics by Castellano et al. (2009). The most intensively studied model is the *voter model*, originally developed by Kimura and Weiss (1964) as a model for spatial conflict of two species (see also Clifford and Sudbury, 1973; Frachebourg and Krapivsky, 1996; Slanina and Lavicka, 2003; Sood and Redner, 2005; Vazquez and Eguíluz, 2008; Schweitzer and Behera, 2008). The analysis of binary opinion models is usually based on mean-field arguments. The microscopic agent configuration is mapped onto an aggregate order parameter, and the system is reformulated on the macro-scale as a differential equation which describes the temporal evolution of that parameter.

A mean-field analysis for the voter model on the complete graph was presented by Slanina and Lavicka (2003), and naturally, we come across the same results using our method. Slanina and Lavicka derive expressions for the asymptotic exit probabilities and the mean time needed to converge, but the partial differential equations that describe the full probability distribution for the time to reach the stationary state is too difficult to be solved analytically (Slanina and Lavicka, 2003, p. 4). Analytical results based on the same methods have been obtained for the voter model on d-dimensional lattices (Cox, 1989; Frachebourg and Krapivsky, 1996; Liggett, 1999; Krapivsky and Redner, 2003) as well as for networks with uncorrelated degree distributions (Sood and Redner, 2005; Vazquez and Eguíluz, 2008).

One step to a more realistic agent description (though still a caricature) is achieved by allowing the agents to make n-ary choices. Among the most popular models that realize this is the Axelrod model (Axelrod, 1997), which uses vectors as state variables and bounded confidence (Hegselmann and Krause, 2002; Deffuant et al., 2001). In both models, the interaction probability is a function of the agent similarity such that similar agents tend to interact and so become more similar in the interaction. In the Axelrod model as well as in other bounded confidence models, this leads to the emergence of clustering as agents converge in homogeneous subgroups while an appropriate distance in between these subgroups increases. An analytical treatment of this process is already quite difficult (see Castellano et al., 2000, for an approximate mean-field analysis). Our Markov chain approach shows how restrictions in the agent communication lead to the emergence of new absorbing states in the associated Markov chain which correspond to system states where different opinions co-exist.

It is however clear that the opinion models we use to illustrate our method are only a caricature of real social processes and important aspects of social behavior and human communication, such as, for instance, preference falsification and indexicality (Kuran, 1997; Garfinkel, 1967) are not taken into account.

The usefulness of the Markov chain formalism in the analysis of more sophisticated ABMs has been discussed by Izquierdo et al. (2009), who look at 10 well-known social simulation models by representing them as a time-homogeneous Markov chain. Among these models are the Schelling segregation model (Schelling, 1971, for which some analytical results are available, for example, in Refs. Pollicott and Weiss, 2001; Grauwin et al., 2010), the Axelrod model (considered above) and the sugarscape model from Epstein and Axtell (1996). The main idea of Izquierdo et al. (2009) is to consider all possible configurations of the system as the state space of the Markov chain. Despite the fact that all the information of the dynamics on the ABM is encoded in a Markov chain, it is difficult to learn directly from this fact, due to the huge dimension of the configuration space and its corresponding Markov transition matrix. The work of Izquierdo and co-workers mainly relies on numerical computations to estimate the stochastic transition matrices of the models.

In our opinion, a well posed mathematical basis for these models may help the understanding of many of their observed properties. Linking the micro-description of an ABM to a macro-description in the form of a Markov chain provides information about the transition from the interaction of individual actors to the complex macroscopic behaviors observed in social systems. In particular, well-known conditions for lumpability make it possible to decide whether the macro model is still Markov. Conversely, this setting can also provide a suitable framework to understand the emergence of long-range memory effects.

In the present stage of this program, due to the obvious limitations of the models we use, the quantitative results obtained can be no more than indications for further analysis of more sophisticated models. We believe however that the strategy and the tools we present here may be useful for future investigation. Moreover we presume that certain aspects pointed by the present analysis, as for instance the behavior in the presence of a large number of agents and the mechanism leading to the emergence of memory effects when passing from the micro to the macro level should also be present in more realistic models.

However, whether there are many empirical applications of either Markov chain approaches (MCA) or ABM, the interweaving of these two lines of research has to be carefully introduced into the empirical setting. Because MCA and ABM were motivated by their own and specific sets of empirical challenges, the simple overlapping of their individual outcomes does not lead to a suitable application framework. The most general subject where both lines ground their common root is structure (or pattern) generation. While in the context of MCA, patterns emerge from time correlations (memory effects) in the lifetime of a single entity or system; in the context of ABM, the emerging structures are mostly related to cross-correlations between individual entities (agents). Several empirical uses have been illustrating the application of either MCA or ABM to solve problems in socio-economic environments. The field of finance is probably the one with the greatest number of empirical applications of both MCA and ABM (Norberg, 2006; Corcuera et al., 2005; Nielsen, 2005; LeBaron, 2000; Cont, 2005). In the former, a macro-description of memory effects usually applies to the identification of business cycles, of periods of stasis and of mutation or economic crashes (Norberg, 2006; Corcuera et al., 2005; Nielsen, 2005). In the latter, agent-based Computational Finance is often used in financial forecasting, such as in the identification of (macro) patterns of collective dynamics from (micro) investor heterogeneity in many financial settings (LeBaron, 2000; Cont, 2005). On the other hand, MCA developed a great variety of methods for testing models from quantitative empirical data. In this sense, while beyond the scope of the present work, we may hope it can contribute to open ABM to be tested when confronted with real situations.

In sociology, the concepts of micro and macro have long been an important subject of analysis. Different but related meanings have been advocated by different authors (see Alexander et al., 1987, and references therein), running from "micro as dealing with individuals and macro as dealing with populations" to "micro as social processes that engender relations among individuals and macro as the structure of different positions in a population and their constraints on interaction". In any case the terms micro and macro relate in this context the action of individuals or small groups based on their mutual relations and the emergence of collective societal scopes.