Social Networks 32 (2010) 212–220

Contents lists available at ScienceDirect

Social Networks

journal homepage: www.elsevier.com/locate/socnet

Bridging: Locating critical connectors in a network

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ARTICLE INFO

Keywords:
Bridges
Bridging
Strength of weak ties
Disease transmission
Behavior change
Opinion leaders

ABSTRACT

This paper proposes several measures for bridging in networks derived from Granovetter's (1973) insight that links which reduce distances in a network are important structural bridges. Bridging is calculated by systematically deleting links and calculating the resultant changes in network cohesion (measured as the inverse average path length). The average change for each node's links provides an individual level measure of bridging. We also present a normalized version which controls for network size and a network-level bridging index. Bridging properties are demonstrated on hypothetical networks, empirical networks, and a set of 100 randomly generated networks to show how the bridging measure correlates with existing network measures such as degree, personal network density, constraint, closeness centrality, betweenness centrality, and vitality. Bridging and the accompanying methodology provide a family of new network measures useful for studying network structure, network dynamics, and network effects on substantive behavioral phenomenon.

The network analysis field has devoted considerable energy developing methods for identifying central nodes in a network which are important to diffusion and other actions that occur on networks (Borgatti and Everett, 2006). In contrast, Granovetter (1973) introduced the concept of bridging which emphasized the importance of structural bridges for diffusion. According to Granovetter (1973, 1982), bridges reduce the overall distance between individuals in a network, enabling information to spread more rapidly throughout a network. The over-emphasis on identifying central nodes has led to the creation of many centrality measures with comparatively less attention given to measures for bridging. Further, most (perhaps all) of the centrality measures developed to date are strongly correlated with a node's degree, its number of direct links.

As Fig. 1 illustrates, there are many centrality measures created to identify important nodes in a network. Degree is a local measure calculated by counting the number of links for each node. Betweenness and closeness (as well as other measures) are global measures calculated using information from the entire network. There is one measure for bridging, constraint, which is calculated using local information only. There are no measures of bridging calculated using complete network information.

In this paper we propose measures for bridging using complete network data that are independent of degree. There are at least three reasons these measures may be useful. First, bridging individuals with few links might act as more efficient diffusion agents than individuals of high degree because they have fewer relationships over which to persuade others (Holme and Ghoshal, 2008). People who are in contact with many others may have less capacity to persuade any one individual because they must spread their persuasive energies across many people, thus diminishing their capacity to persuade any one person. A critical node with few others to persuade may devote more energy to persuading those others and hence be a more effective change agent.

Second, bridging individuals may be more receptive to behavior change and more likely to be persuaded by targeted communications. Individuals with high degree occupy prominent and visible positions in the network. This prominence can inhibit behavior change because prominent individuals need to support the status quo in order to maintain their positions of prominence (Becker, 1970; Cancian, 1979). Bridging individuals, in contrast, have fewer direct contacts and therefore less direct pressure to support prevailing norms and behaviors, and hence perhaps more susceptible to change.

Finally, it may be that occupying a bridging position is indicative of attitudinal and behavioral dispositions such as being open to new ideas and practices. Many studies have identified associations between degree and attitudes and behaviors. Degree is often equated with opinion leadership and many studies conducted to determine correlates of opinion leadership (Rogers, 2003). Similarly, it is reasonable to expect that there may be attitudinal and behavioral correlates of bridging. Burt (1992) has shown that spanning structural holes accrues advantages to managers. We suspect that the graph theoretic measures of bridging proposed here will

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doi:10.1016/j.socnet.2010.03.003

		Measure	
		Centrality	Bridging
Network Information	Local	Degree	Constraint
	Global	Closeness Betweenness	Bridging based on link deletions

Fig. 1. Existing nodal measures of structural position can be classified by whether they measure centrality or bridging and whether they use local or complete information.

also be associated with such advantages, or with other individual characteristics.

In sum, bridging individuals may be more effective at changing others, more open to change themselves, and intrinsically interesting to identify. The efficacy of and susceptibility to behavior change of bridging individuals may be a function of the innovation's attributes such as its cultural or normative compatibility. Innovations that are radically new, less compatible with cultural norms, or have the potential to change power dynamics within a community or organization may be more readily embraced by bridging individuals than leaders because leaders have a vested interest in maintaining the status quo.

To measure bridging, Burt developed the concept of structural holes and argued that individuals who spann structural holes form bridges in the network (Burt, 1992). This spanning function was measured by constraint which is the degree a person's links (ego network) are to people not connected to one another. Constraint calculates bridging using network data from the individual's local or personal network rather than considering the structure of the complete network. Given the importance of bridging behavior to interpretation of network structure and diffusion, it seems warranted to develop measures of bridging based on complete network information.

Doreian and Fujimoto (2004) proposed three methods for identifying linking-pin organizations of (1) blockmodeling, (2) centrality/centralization, and (3) cut-points/sets of the graph. By using empirical data, they found that the necessary (but not sufficient) condition for a node to be a linking-pin organization is that it be a singleton in a position of a blockmodel image network, and further if it is a cut-vertex in the image network, it is a strong linking-pin organization.

In graph theory, two concepts have been used to describe bridging. A cut-point is a node whose removal disconnects a network and a bridge is a link whose removal disconnects the network (Harary et al., 1965). These measures (cut-point and bridge) have only been used to identify one or few nodes and links in a network and do not provide individual measures that can be used in subsequent behavioral analyses. Further, these two measures are very limited definitions of the much broader concept of bridging. The bridging measures described in this paper calculate the change in average path length of the network when each link is removed. These values are then summarized for each node. Before presenting the mathematical derivation of the measures, we provide some background on the use of node and link deletion in networks.

Link and node deletion. Many researchers have used link deletion for blockmodeling and subgroup identification (Everett, 1983; Schwartz and Sprinzen, 1984; Borgatti et al., 1990). Another use of link deletion and addition has been in the analysis and measurement of small worlds (Watts, 1999). Bridges make networks small world networks by reducing the overall path length between nodes in a network. Another example of link deletion is the Girvan and Newman (2002) procedure for removing links and recalculating network properties to define community structure. Motter et al. (2002) removed selected links from various prototypical networks to demonstrate network vulnerability. White and Harary (2001) and Moody and White (2003) proposed deleting links to assess the overall cohesiveness of a network. These link deletion analyses remove links based on some criterion and then calculate a network-level property, typically until an optimal or desired level of some network-level outcome is reached.

In addition to link deletion, some researchers have deleted nodes to assess their importance. Koschützki et al. (2005) proposed a vitality measure calculated by removing nodes and calculating change in closeness centrality (Koschützki et al., 2005, p. 36). Similarly, flow betweenness is calculated by identifying the flow through a node divided by total flows in the network with that node removed (Freeman et al., 1991). Borgatti's (2006) Key Player concepts and algorithms also use node removal to find sets of nodes that optimally span the network. Node deletion measures (vitality, flow betweenness, key players) differ from the bridging measures proposed in this paper since they use node removal not link removal. Link removal is quite distinct and more versatile than node removal.

The present approach differs from these prior techniques in at least two ways: (1) all links in the network are systematically removed, and (2) the resultant change in a network-level measure is used to characterize nodes rather than the link or network. Conceptually the proposed measure is similar to Borgatti's (2006) Key Player analysis with the difference being that Key Player uses node deletion while the bridging measures proposed here use link deletion and then aggregates the changes to the nodes. The main contribution of the present approach is that it provides an individual measure of the strategic function of a node's links. The calculation involves taking the average of each node's link changes, not the sum, and this average captures the importance of each person's position; and unlike other positional measures is independent of the node's degree.

The measure. As in other studies (Borgatti, 2006), the network property of interest is the overall cohesion in the network defined as (Freeman, 1979):

$$C = \frac{\sum 1/d_{ij}}{N(N-1)} \quad (i \neq j) \tag{1}$$

where d_{ij} is the geodesic distance between dyads. Disconnected dyads are given a value of infinity, and therefore, $1/d_{ij}$ is equal to $1/\infty$ which reduces to zero. For the purposes of this derivation, it is useful to rewrite the common matrix representation of cohesion into its row/column constituent elements:

$$C = \frac{\sum_i^N \sum_j^{N-1} 1/d_{ij}}{N(N-1)} = \frac{1}{N}\sum_i^N \frac{1}{N-1}\sum_j^{N-1} \frac{1}{d_{ij}} \tag{2}$$

The value of the reciprocal of the geodesic distance ranges from 0 to 1, assuming unreachable nodes are infinitely separated. Thus, the maximum value of the inner sum is unity (the $N-1$ reflects the fact that the node cannot have a link to itself). Likewise, the outer (leftmost) sum has a maximum possible value of unity. To calculate change from link deletion we have:

$$\Delta C_{ij} = C - C_{ij'} = \left(\frac{1}{N}\sum_i^N \frac{1}{N-1}\sum_j^{N-1}\frac{1}{d_{ij}}\right) - \left(\frac{1}{N}\sum_i^N \frac{1}{N-1}\sum_j^{N-1}\frac{1}{d_{ij'}}\right)$$
$$= \frac{1}{N}\sum_i^N \frac{1}{N-1}\sum_j^{N-1}\left(\frac{1}{d_{ij}} - \frac{1}{d_{ij'}}\right) \tag{3}$$

where C is an overall cohesion, $C_{ij'}$ is the cohesion of the network without a link from i to j, and ΔC_{ij} the difference in cohesion when the link from i to j is removed. Given that the removal of an existing link increases (or leaves unchanged) the geodesic distance to the remaining connected nodes, this implies that the result of ΔC_{ij} is zero or a manifestly positive number. Furthermore, the upper limit of ΔC_{ij} is unity, since the limit of C is also unity.