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Speed or spacing? Cumulative variables, and convolution of model errors and time in traffic flow models validation and calibration

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ABSTRACT

This paper proves that in traffic flow model calibration and validation the cumulative sum of a variable has to be preferred to the variable itself as a measure of performance. As shown through analytical relationships, model residuals dynamics are preserved if discrepancy measures of a model against reality are calculated on a cumulative variable, rather than on the variable itself. Keeping memory of model residuals occurrence times is essential in traffic flow modelling where the ability of reproducing the dynamics of a phenomenon – as a bottleneck evolution or a vehicle deceleration profile – may count as much as the ability of reproducing its order of magnitude. According to the aforesaid finding, in a car-following models context, calibration on travelled space is more robust than calibration on speed or acceleration. Similarly in case of macroscopic traffic flow models validation and calibration, cumulative flows are to be preferred to flows. Actually, the findings above hold for any dynamic model.

1. Introduction

The measures of discrepancy between a simulation and the real world are at the core of any methodology aimed at reducing the uncertainty involved in scientific modelling, as model calibration and validation (in traffic simulation calibration and validation constitute the object of substantial research efforts see e.g. the EU COST Action TU0903-MULTITUDE and related outcomes, Daamen et al., 2013; Brackstone and Punzo, 2014, or the starting TRB's Task Force on Transportation System Simulation; AHB80T, 2015).

As for dynamic models, discrepancy is mainly measured on the interest variables time-series. With this kind of measure, the ability of a dynamic model to reproduce the temporal evolution of a system can be observed. Several error statistics – also known as 'global error statistics' – are generally used to quantify such discrepancy, or degree of match, between simulated and measured time-series. Examples include the sum of squared or absolute errors and all their linear transformations, and Theil's inequality coefficient (for a review about their use in traffic modelling, see Hollander and Liu, 2008; Brackstone and Punzo, 2014; Buisson et al., 2014).

In global error statistics the simulated value of an interest variable at each instant is compared with its measured value at the same instant. However, the temporal evolution of model residuals and its features, among them the autocorrelation

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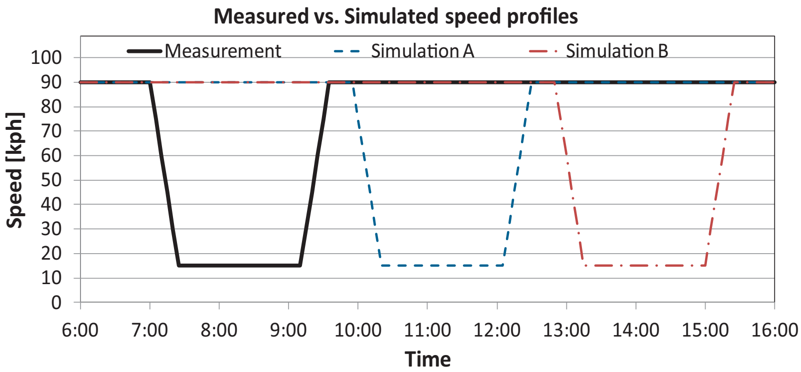

Fig. 1. Comparison between a measured speed time-series ("Measurement") and two simulated speed time-series ("Simulation A" and "Simulation B").

of residuals, do not play any role and, therefore, do not affect the calculation of the error statistic. It is only the magnitude of residuals that matters, not their occurrence time or sequence. For instance, if the sum of squared residuals has to be minimized in calibration, the minimization will be driven by the residuals with the highest magnitude (the residuals being squared) irrespective of their occurrence time. This is a major drawback in dynamic traffic models validation too, where the model ability to reproduce the duration of an event, as a bottleneck, may count as much as the ability to capture its magnitude.

This issue is illustrated in Fig. 1. It shows a time-series of measured speed data and two time-series of simulated speed data. It provides a qualitative representation of time-series of vehicle mean speeds at a spot detector, or of vehicle speed profiles (on a different time scale). The measured speed profile exhibits a drop in the period 7:00–9:30 a.m. while the two simulated speed profiles i.e. Simulation A and Simulation B exhibit a drop that is equal to the drop in the measured profile as for magnitude, but it is shifted in time (for simulations A and B the speed drop occurrence period is respectively 10:00 a.m. to 12:30 p.m. and 1:00–3:30 p.m.). It is easily understood that any of the above mentioned error statistics would yield the same value for both the simulated profiles, as those statistics have no memory of the residuals occurrence time. Actually profiles A and B are very different from one another, and A is closer to the measured profile than B.

With regards to model residuals autocorrelation, in particular, Hoogendoorn and Hoogendoorn (2010) acknowledged that it affects microscopic traffic flow model calibration. To avoid biased calibration results, they suggest an a posteriori transformation of measured and simulated time-series in the 'time-domain' that is proved to eliminate autocorrelation in case of linear models. Montanino et al. (2012) pointed instead at the aforementioned weakness of global error statistics in the time-domain suggesting to conceive error statistics in the 'frequency-domain'. Frequency-domain statistics make the most of the information about residuals autocorrelation in time-series data. Yet in a frequency-domain approach the magnitude of local errors is not directly taken into account.

Therefore, a feasible approach to overcome the issue raised (in the time-domain), i.e. keeping memory of the model residuals order, might be assigning weights to the residuals. In a systematic way, this can be obtained through a convolution of residuals and time. This is the basic idea inspiring this paper.

In general, given two functions f and g, their convolution is defined as the integral of their product after one is reversed and shifted (Damelin and Miller, 2011):

$$(f*g)(t) = \int_{-\infty}^{+\infty} f(\tau)g(t-\tau)d\tau = \int_{-\infty}^{+\infty} f(t-\tau)g(\tau)d\tau$$

In our case, the discrete convolution of the model residuals time-series $g[k]$ and the time k, in the time interval $[1, N]$, can be written as:

$$(f*g)[h] = \sum_{k=1}^{N}[h-k]g[k]$$

For a shift h equal to the total length N of the time-series the previous equation yields:

$$(f*g)[N] = \sum_{k=1}^{N}[N-k]g[k] \tag{1}$$

The discrete convolution in Eq. (1) represents the linear combination of the model residuals with the reversed discrete time. The first residual is weighted with the length N of the time-series, while the last residual is weighted with one. It is easily understood that the time convolution above allows preserving the memory of model residuals dynamics. The mathematical operation in Eq. (1) is therefore a possible method to overcome the aforementioned weakness of global error statistics.