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On the flexibility of using marginal distribution choice models in traffic equilibrium☆

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ABSTRACT

Traffic equilibrium models are fundamental to the analysis of transportation systems. The stochastic user equilibrium (SUE) model which relaxes the perfect information assumption of the deterministic user equilibrium is one such model. The aim of this paper is to develop a new user equilibrium model, namely the MDM-SUE model, that uses the marginal distribution model (MDM) as the underlying route choice model. In this choice model, the marginal distributions of the path utilities are specified but the joint distribution is not. By focusing on the joint distribution that maximizes expected utility, we show that MDM-SUE exists and is unique under mild assumptions on the marginal distributions. We develop a convex optimization formulation for the MDM-SUE. For specific choices of marginal distributions, the MDM-SUE model recreates the optimization formulation of logit SUE and weibit SUE. Moreover, the model is flexible since it can capture perception variance scaling at the route level and allows for modeling different user preferences by allowing for skewed distributions and heavy tailed distributions. The model can also be generalized to incorporate bounded support distributions and discrete distributions which allows to distinguish between used and unused routes within the SUE framework. We adapt the method of successive averages to develop an efficient approach to compute MDM-SUE traffic flows. In our numerical experiments, we test the ability of MDM-SUE to relax the assumption that the error terms are independently and identically distributed random variables as in the logit models and study the additional modeling flexibility that MDM-SUE provides on small-sized networks as well as on the large network of the city of Winnipeg. The results indicate that the model provides both modeling flexibility and computational tractability in traffic equilibrium.

1. Introduction

Traffic equilibrium models are fundamental to the analysis of transportation systems. The inputs of a traffic model are the topology of the traffic network, origin-destination (OD) pairs, demands of OD pairs and the link performance functions that take congestion, travel distance and road pricing into account. The goal of the model is to estimate the link and route flows in the traffic network.

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A reasonable assumption underlying many of the models of traffic flow is that the users try to choose routes that maximize their utilities. The utility of a user choosing a route depends on the travel cost of the route which is calculated using the performance functions of the links that are included in the route. The equilibrium state can be regarded as the state where no traveler has incentive to change their route choices. The concept of the user equilibrium (UE) was first proposed by Wardrop (1952) and defined as the traffic state at which journey times on all routes which are actually used are equal or less than those which would be experienced by a single vehicle on any unused route. Therefore, no user would reduce their travel costs by unilaterally changing their routes.

The stochastic user equilibrium (SUE) relaxes the perfect information assumption and is a generalization of the concept of UE. Daganzo and Sheffi (1977) defined the SUE as the traffic state at which no user can improve his/her perceived travel time by unilaterally changing routes. From a modeling perspective, the SUE introduces a random perception error term to the utility function. The statistical distribution of the perception error term defines the underlying discrete choice model and the route choice behavior of the users. Burrel (1968) proposed to use a uniform distribution. Two of the more popular distributions used in discrete choice models are Gumbel and normal distributions. Dial (1971) proposed the use of the multinomial logit (MNL) route choice model in traffic assignment which is defined by independent and identically distributed (i.i.d) Gumbel distributed error terms. Computational efficiency is crucial in traffic assignment and hence the MNL model has received a lot of attention since it provides closed form choice probabilities. On the other hand, two assumptions of the MNL model that have received particular criticism in the traffic assignment literature (Sheffi, 1985) are:

(a) Random terms are independently distributed.
(b) Random terms are identically distributed.

Routes share links and it is desired to model the effect of overlapping routes on choices of the users. Moreover, users may have different perception variances for routes and relaxing the equal variance assumption (a consequence of the identical distribution assumption above) provides greater flexibility in modeling user behavior. Many researchers have focused on relaxing the independence assumption above and proposed variants of the MNL model (see Chu, 1989; Cascetta et al., 1996; Bekhor and Prashker, 1999; Ben-Akiva and Bierlaire, 1999; Zhou et al., 2012). In a recent study, Chen et al. (2012) proposed an approach that scales the logit dispersion parameters at the OD level. The underlying motivation is that the longer routes are subject to greater perception errors. The standard logit models do not allow to scale the variances in the route level, and hence, all the routes belonging to the same OD pair need to have the same perception variance. Castillo et al. (2008) proposed an alternative approach to relaxing the equal variance assumption by using Weibull distributed random error terms. In their proposed model termed as the multinomial weibit (MNW) model, the choice probabilities are in closed form with route perception error variances defined as functions of the route travel times. This can be regarded as scaling the variances at the route level. Recently, Kitthamkesorn and Chen (2013) have extended the MNW model by adding a correction term to simultaneously deal with the overlapping routes and the equal variance assumption.

As an alternate, the multinomial probit (MNP) model was proposed by Daganzo and Sheffi (1977) which uses normally distributed random error terms and does not inherit the independence and equal variance assumptions. It provides greater modeling flexibility since it allows to assign route-specific perception error variances which do not necessarily depend on the route lengths or travel costs. However, MNP does not have a closed form expression and its solution requires significant computational effort. In a recent paper, Ahipasaoglu et al. (2015) proposed a new SUE model referred to as the cross moment-stochastic user equilibrium model (CMM-SUE) which relaxes the assumption of normality for the distribution of the error terms. The underlying choice model uses only the mean and covariance information of route utilities. The choice probabilities are computed for the joint distribution of path utilities that maximizes expected utility of users. The CMM-SUE flow is obtained by solving a convex optimization problem as an alternative to the simulation based MNP model. Their approach however does not incorporate additional information on the marginal distributions, such as skewness or heavy tails of the distribution.

In this study, we introduce an SUE model that is based on a recently proposed discrete choice model—the marginal distribution model (MDM; Natarajan et al., 2009; Mishra et al., 2014). The major difference of the MDM-SUE model from the aforementioned models is that it assumes the knowledge of marginal distributions of the error terms but does not assume that the random error terms are independently or identically distributed. Instead, the choice model is based on the joint distribution that maximizes the expected utility for users. Being able to incorporate information on the marginal distributions into the choice model makes it very flexible as it can model a wide range of route choice behavior. The main contributions of the paper are summarized next:

(a) We propose a convex optimization formulation to compute the traffic flow in the MDM-SUE model. This formulation is an extension of the entropy type formulation of Fisk (1980) for the logit model. We also show that under mild assumptions on the marginal distributions, the MDM-SUE exists and is unique. We show that the MDM-SUE flow values can be interpreted as the solution to a robust optimization problem where the system planner accounts for the worst possible distribution from a set of distributions for the error terms of the routes.
(b) We show that for specific choices of marginal distributions, MDM-SUE reduces to the logit-based and weibit-based SUE models. Moreover, the flexibility of MDM-SUE in terms of specifying the marginal distribution allows to generalize the recently proposed hybrid logit-weibit SUE model of Xu et al. (2015) to other distributions. Most existing models in literature use unbounded distributions for the random utilities for which a positive probability is assigned to every