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Traffic state estimation through compressed sensing and Markov random field

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ABSTRACT

This study focuses on information recovery from noisy traffic data and traffic state estimation. The main contributions of this paper are: *i*) a novel algorithm based on the compressed sensing theory is developed to recover traffic data with Gaussian measurement noise, partial data missing, and corrupted noise; *ii*) the accuracy of traffic state estimation (*TSE*) is improved by using Markov random field and total variation (*TV*) regularization, with introduction of smoothness prior; and *iii*) a recent *TSE* method is extended to handle traffic state variables with high dimension. Numerical experiments and field data are used to test performances of these proposed methods; consistent and satisfactory results are obtained.

1. Introduction

As an important topic in intelligent transportation systems, traffic state estimation (*TSE*) typically consists of three phases (implicitly or explicitly): data denoising and recovery, traffic model simplification, and model-based traffic state estimation. However, most of the existing *TSE* algorithms suffer the following issues:

First, in the data denoising phase, previous studies usually assume a Gaussian distribution for data noise (Wang and Papageorgiou, 2005; Sun et al., 2003; Laval et al., 2012; Deng et al., 2013), and ignore corrupted noise and partial data missing. Corrupted noise and partial missing are not rare in traffic data, and difficult to be properly handled by commonly used denoising methods (e.g., moving average, Fourier transform, etc.). Advanced signal processing techniques have been utilized to handle corrupted noise and partial data missing separately. For example, the corrupted noise was interpreted as the abrupt changes in traffic time series and handled through a multi-resolution wavelet approach (Zheng et al., 2011a; 2011b; Zheng and Washington, 2012); Wang and Papageorgiou (2005) considered partial missing using an auxiliary random walk assumption within an extended Kalman filtering framework. To the best of our knowledge, no *TSE* study has considered Gaussian noise, corrupted noise and partial missing simultaneously.

Meanwhile, in the phase of traffic model simplification, to make nonlinear traffic models tractable, existing TSE methods often reduce the complexity of the models at the cost of the models' accuracy and their numerical stability, e.g., Wang and Papageorgiou (2005) approximated a second order macroscopic traffic model by its first order Taylor series expansion. Muñoz et al. (2003) transformed the nonlinear (minimization) state equations of the modified Cell Transmission Model

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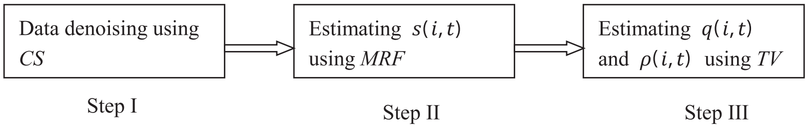

Fig. 1. The flow diagram of the proposed method of traffic state estimation.

(*MCTM*) into a set of piecewise linear equations through Monte Carlo simulations. More recently, Sumalee et al. (2011) extended *MCTM* and proposed the Stochastic Cell Transmission Model (*SCTM*). Specifically, five operational modes corresponding to different congestion levels of a freeway segment are considered, and their overall effect is estimated based on the theory of finite mixture distribution. Their estimated density is reported to be more accurate than that from *MCTM* in terms of variance, although the two models perform similarly in terms of the mean value of the estimated densities.

Finally, related to the first shortcoming previously discussed, Kalman filtering theory is often applied in the existing *TSE* methods to generate traffic state estimations; however, the measurement noise and the process noise of the state vector in the Kalman filtering framework are restricted as Gaussian noise, and such restriction can smooth out information-rich discontinuities (singularities) in the original traffic data (Zheng and Washington, 2012). Furthermore, applying Kalman filtering techniques can be computationally demanding, especially when the dimension of the estimated traffic state variables is high. For example, Deng et al. (2013) recently proposed a *TSE* method based on Kalman filtering with Clark's approximation (*CAKF*). The memory requirement of this algorithm is quadratic to the traffic state dimension and the time complexity is cubic to the traffic state dimension. More detail on this algorithm is provided in Appendix B.

The issues above motivated this study. The main objective of this paper is to improve *TSE* by incorporating advanced techniques from other fields at each phase of *TSE*, namely, compressed/compressive sensing (*CS*) for data denoising and information recovery, Markov random field (*MRF*) for helping to achieve linearization of traffic flow model, and a total variation (*TV*) regularization for estimating traffic states. In addition, we also improve a recently developed *TSE* method (Deng et al., 2013) to estimate traffic states with high dimension by using the Schur decomposition (Boyd and Vandenberghe, 2009).

As shown in Fig. 1, the proposed methodology consists of three steps. At Step I, the input traffic signal is preprocessed by a *CS*-based information recovery method to diminish the effects of data missing, corrupted noise and Gaussian noise; at Step II, a method using *MRF* is developed to estimate traffic state $s(i, t)$; at Step III, traffic flow $q(i, t)$ and density $\rho(i, t)$ are estimated through a *TV* scheme.

The structure of the remaining of this paper follows: Section 2 reviews the general theories and algorithms in *CS* for data denoising and information recovery, and then proposes our algorithm and theoretical result for applying *CS* in the context of *TSE*; Section 3 reviews notable existing *TSE* algorithms and then proposes a MRF-based approach for traffic model simplification and a *TV* regularization for traffic state estimation; Section 4 presents test results for data denoising as well as traffic state estimation, based on numerical experiments and field data; finally, Section 5 concludes the paper by summarizing main findings and discussing future research needs.

Note that unless stated otherwise, in this paper data and measurement are exchangeable, noise means a disturbance in the measurement that obscures the measurement quality, and error is used to indicate accuracy, which means the difference between the generated value and the true value.

2. Data recovery through CS

Below are notations that are used in this section. Note that the list of notations in the beginning of each major section of this paper is not exhaustive. To be concise, some notations that are unlikely to cause confusion are not included on the lists.

$\mathrm{q(t)}$	the true value of traffic flow at time t
$\mathrm{\hat{q}(t)}$	the measurement flow value at time t.
$\check{\mathrm{q}}$	Fourier transform of q.
$\hat{\mathrm{q}}_{\wedge}$	$\lvert\wedge\rvert$-dimensional sub-vector of $\hat{\mathrm{q}}$ with elements indexed by $\wedge$.
$\wedge$	the random subset of time indices of non-missing measurements.
Λ^c	the time indices set of missing measurements
x	n-dimensional vector to be recovered.
$\hat{\mathrm{x}}$	the solution of minimize $\lVert\mathrm{x}\rVert_1$ s.t. $\lVert\mathrm{Ax}-\mathrm{b}_2\rVert \le \eta$.
$\sigma_k(\mathrm{x})_1$	the ℓ_1 residual norm of the best k-sparse approximation of x.
$\varepsilon_G(\mathrm{t})$	Gaussian noise at time t.
$\varepsilon_s(\mathrm{t})$	the corrupted noise at time t.
$\varepsilon_{G\lvert\wedge}$	$\lvert\wedge\rvert$-dimensional sub-vector of ε_G with elements indexed by $\wedge$.
$\varepsilon_{S\lvert\wedge}$	$\lvert\wedge\rvert$-dimensional sub-vector of ε_s with elements indexed by $\wedge$.
A	$m \times n$ measurement matrix.
b	m-dimensional measurement vector.