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Robust intermodal hub location under polyhedral demand uncertainty

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A R T I C L E I N F O

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A B S T R A C T

In this study, we consider the robust uncapacitated multiple allocation p-hub median problem under polyhedral demand uncertainty. We model the demand uncertainty in two different ways. The hose model assumes that the only available information is the upper limit on the total flow adjacent at each node, while the hybrid model additionally imposes lower and upper bounds on each pairwise demand. We propose linear mixed integer programming formulations using a minmax criteria and devise two Benders decomposition based exact solution algorithms in order to solve large-scale problems. We report the results of our computational experiments on the effect of incorporating uncertainty and on the performance of our exact approaches.

1. Introduction

Hubs are facilities that consolidate and distribute flow from many origins to many destinations. Hub structure is common in transportation networks that benefit from economies of scale such as airline and cargo delivery networks. Many variants of hub location problems have been studied in the last few decades. The p-hub median problem is one of the most studied problems in the hub location literature. In the p-hub median problem, the aim is to locate p hubs and to route the flow between origin-destination pairs through these hubs so that the total transportation cost is minimized. Direct shipments between nonhub nodes are usually not allowed. There are variants of the problem where a nonhub node can send and receive traffic through all hubs and others where there is a restriction on the number of hubs that a nonhub node can be connected to. The former is known as the multiple allocation setting. In some other variants, hub or edge capacities are imposed. In this paper, we study an uncapacitated p-hub median problem with multiple allocation and no direct shipments. In the p-hub median problem, the routing cost between two hub nodes is discounted independently of the amount of flow travelling between these two hubs. For this reason, this problem may not model the discounts due to economies of scale correctly. On the other hand, it has applications in intermodal transportation where discounts on hub-to-hub transfers apply due to the use of a cheaper transportation mode such as rail or maritime transportation.

An important issue that arises while designing a hub network is coping with the uncertainty in the data. The p-hub median problem is solved in the strategic planning phase, usually before actual point-to-point demand values are realized and the network starts operating. The demand may have large variations depending on the seasons, holidays, prices, level

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of economic activities, population, service time and quality and the price and quality of the services provided by the competitors. A decision made based on a given realization of the data may be obsolete in time of operation.

The uncertainty in the demand values can be modeled in various forms: (i) the probability distribution of demand values may be known; (ii) the probability distribution may not be known but demands can take any value in a given set; (iii) a discrete set of possible scenarios may be identified. In this study, we model uncertainty with a polyhedral set. More precisely, we consider the hose model and its restriction with box constraints. The hose model has been introduced by Duffield et al. (1999) and Fingerhut et al. (1997) to model demand uncertainty in virtual private networks. In the hose model, the user specifies aggregate upper bounds on inbound and outbound traffic of each node. Modeling uncertainty with this model has several advantages. First, it is simpler to estimate a value for each node compared to for each node pair. Second, it has resource-sharing flexibility. Third, it is less conservative compared to a model in which each origin-destination demand is set to its worst case value. Finally, it has the advantage of reducing statistical variability through aggregation. Still, the hose model contains extreme scenarios in which few origin destination pairs may have large traffic demands and remaining pairs may have zero traffic. To consider more realistic situations, Altın et al. (2011a) propose to use a hybrid model where lower and upper bounds on individual traffic demands are added to the hose model. This requires estimation of bounds for each node pair but leads to less conservative solutions. These uncertainty models are suitable for transportation applications where pairwise demands are often estimated based on factors such as the population, level of economic activity and access to transportation infrastructure at origins and destinations (see, e.g., Bhadra, 2003 who examines the relationship between origin and destination travel and local area characteristics and Hsiao and Hansen, 2011). The hose model is a simple way of modeling correlations such as a person flying from Istanbul to Paris is not flying at the same time from London to Istanbul.

To hedge against uncertainty in the demand data, we adopt a minmax robustness criterion and minimize the cost of the network under the worst case scenario. In robust optimization, commonly, one does not make assumptions about the probability distributions, rather assumes that the data belongs to an uncertainty set. A robust solution is one whose worst case performance over all possible realizations in the uncertainty set is the best (see, e.g., Atamtürk, 2006; Ben-Tal et al., 2004; Ben-Tal and Nemirovski, 1998, 1999, 2008; Bertsimas and Sim, 2003, 2004; Mudchanatongsuk et al., 2008; Ordóñez and Zhao, 2007; Yaman et al., 2001, 2007b).

In this study, we introduce the robust multiple allocation p-hub median problem under hose and hybrid demand uncertainty. Our contribution is to incorporate demand uncertainty into a classical problem and to investigate the gain of recognizing the uncertainty. We derive mixed integer programming formulations and propose exact solution methods based on Benders decomposition. In our computational experiments, we first analyze the changes in cost and hub locations with different uncertainty sets. Then we test the limits of solving the model with an off-the-shelf solver and compare the performances of two decomposition approaches. Our computational experiments showed that the decomposition algorithms are able to solve large instances that cannot be solved with an off-the-shelf solver and that it is possible to obtain significant cost savings in case of demand fluctuations by incorporating uncertainty into the decision making process.

The rest of the paper is organized as follows. In Section 2, we review the related studies in the literature. In Section 3, we introduce the robust multiple allocation p-hub median problem under hose and hybrid demand uncertainty and propose mixed integer programming formulations. We devise two different Benders decomposition based exact solution algorithms in Section 4 and report our computational findings in Section 5. We conclude in Section 6.

2. Literature review

Hub location has grown to be an important and well-studied area of network analysis. Detailed surveys of studies on hub location are given in Campbell (1994b), O'Kelly and Miller (1994), Klincewicz (1998), Campbell et al. (2002), Alumur and Kara (2008), Campbell and O'Kelly (2012) and Farahani et al. (2013).

Here we review first the studies on the uncapacitated multiple allocation p-hub median problem (UMApHMP) and then the studies on hub location problems under data uncertainty.

UMApHMP is first formulated by Campbell (1992). Alternative formulations with four index variables are given by Campbell (1994a) and Skorin-Kapov et al. (1996). Ernst and Krishnamoorthy (1998a) propose a three-indexed formulation based on aggregated flows. Various exact and heuristic solution algorithms are devised to solve UMApHMP efficiently (see, e.g., Campbell, 1996; Ernst and Krishnamoorthy, 1998a; 1998b). Besides, the variant of the problem where the number of hubs is not fixed, namely the uncapacitated multiple allocation hub location problem with fixed costs (UMAHLP), is studied by Campbell (1994a), Klincewicz (1996), Ernst and Krishnamoorthy (1998a), Ebery et al. (2000), Mayer and Wagner (2002), Boland et al. (2004), Hamacher et al. (2004), Marín (2005), Cánovas et al. (2007) and Contreras et al. (2011a). Since this problem is analogous to the UMApHMP, most of the solution methods can be adapted to solve the UMApHMP.

Several Benders decomposition based approaches have been proposed to solve the uncapacitated multiple allocation hub location problems and they proved to be effective. To the best of our knowledge, Camargo et al. (2008) are the first ones to apply Benders decomposition to the uncapacitated multiple allocation hub location problem. They propose three different Benders approaches. The first one is the classical approach, which adds a single cut at each iteration, while the second is the multi-cut version in which Benders cuts are generated for each origin-destination pair. Another variant allows an error margin ϵ for the cuts added and the algorithm terminates when an ϵ-optimal solution is obtained. They solve instances with up to 200 nodes and conclude that the single-cut version of the algorithm shows the best computational performance. Contreras et al. (2011a) propose a Benders decomposition algorithm to solve UMAHLP. They generate cuts for each candidate