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# The electric vehicle touring problem

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ABSTRACT

The increasing concern over global warming has led to the rapid development of the electric vehicle industry. Electric vehicles (EVs) have the potential to reduce the greenhouse effect and facilitate more efficient use of energy resources. In this paper, we study several EV route planning problems that take into consideration possible battery charging or swapping operations. Given a road network, the objective is to determine the shortest (travel time) route that a vehicle with a given battery capacity can take to travel between a pair of vertices or to visit a set of vertices with several stops, if necessary, at battery switch stations. We present polynomial time algorithms for the *EV shortest travel time path problem* and the *fixed tour EV touring problem*, where the fixed tour problem requires visiting a set of vertices in a given order. Based on the result, we also propose constant factor approximation algorithms for the *EV touring problem*, which is a generalization of the traveling salesman problem.



## 1. Introduction

Transportation is one of the fastest-growing sources of greenhouse gas emissions that contribute to climate change. In the United States, transportation accounts for approximately 25 percent of total greenhouse gas emissions (U.S. EPA Environmental Protection Agency, 2012). Consequently, during the last decade, the automobile industry has developed an increasing number of electric (battery) vehicles or hybrid electric vehicles to deal with the rising cost of energy. Electric vehicles (EVs), which release almost no air pollutants, could make a significant contribution to maintaining the quality of the environment. The Electric Power Research Institute estimates that EVs will account for 6%–30% of the vehicles in use by 2030 (Electric Power Research Institute, 2009).

An efficient EV routing service would obviously encourage the transition to electric vehicle use. The U.S. Department of Energy has developed an online service (U.S. DOE Department of Energy, 2012) that provides a route map interface, as well as information about EV charging facilities for EV owners. However, it is very difficult to design an optimal EV route planning service because EVs have some serious limitations. The first is the low energy capacity of batteries. Currently, their range is only 150 to 200 kilometers; hence, EVs are used primarily in urban areas. The second problem is that EV batteries require a long charging time. At the moment, they can be fully recharged from empty in 2 to 6h, depending on the level of charging available at the station. These factors have delayed the growth of the EV market; however, EVs can now be refueled in a

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matter of minutes through a system called battery-swaps. Recently, Tesla Motors (Tesla Motors, 2013) provided the solution via a network of battery switch stations. The state-of-the-art technology leads to a new model of EV route planning.

In this paper, we explore some interesting models that incorporate the battery capacity constraint when an electric vehicle is driven. First, we begin with the *EV shortest travel time path problem*. In this problem, we determine a route from a source to a destination that an electric vehicle with a given battery capacity $U$ can travel along so that the total time including traveling and battery-swaps is minimized. If necessary, the vehicle can stop at several battery switch stations on the route to maintain its movement. Note that we measure our objective in terms of time. That is, the weight of an edge represents the time required for the vehicle to travel through the edge, and the capacity represents the length of time the vehicle can travel with a full battery. Similar to the traveling salesman problem (TSP), the *EV touring problem* involves organizing a tour of a set of cities so that the total time required is minimized. The vehicle visits each city and returns to the origin, stopping at battery switch stations whenever necessary. We consider two scenarios: the *on-site station* and the *off-site station* models in which each city has an *on-site* battery switch station and an *off-site* battery switch station within an acceptable distance, respectively.

**Our contribution.** The main results obtained in this paper are summarized as follows:

1. We consider the EV shortest travel time path problem and present a simple dynamic programming algorithm that runs in $O(kn^2)$ time, where $k$ is a given upper bound of the number of battery-swaps and $n$ is the order of the graph.
2. We develop efficient polynomial time algorithms for the fixed tour EV touring problem, where the fixed tour constraint requires visiting a set of cities and returning to the origin in a given order. This result extends the previous studies of the *fixed path gas station problem* reported in Khuller et al. (2011), Lin (2008), Lin et al. (2007) by using graph-theoretic techniques.
3. We propose two approximation algorithms within a $\frac{9}{4}$-factor and a $\frac{9}{2}$-factor, respectively, for the uniform and non-uniform cost on-site station EV touring problem. Moreover, if the battery capacity is sufficiently large, the approximation ratio is the same as that of the well-known Christofides algorithm for the TSP, i.e., $\frac{3}{2}$.
4. We also study the off-site station EV touring problem and propose a $\frac{3}{2}(\frac{3+2\alpha}{1-2\alpha})$-approximation algorithm to solve the problem, where $\alpha$ is a given acceptable distance between a city and its nearest battery switch station.

## 2. Preliminaries

A great deal of research has been devoted to the shortest route planning problem; and many variations and extensions of the problem have been proposed. One related problem is the well-known *capacitated vehicle routing problem*, which involves finding a set of routes that begin at a depot, visit multiple customers and deliver goods, and return to the depot such that the number of vehicles, each of which has a limited carrying capacity, is minimized or the total distance is minimized. Readers may refer to Laporte's survey (Laporte, 2009) and Pillac et al.'s review (Pillac et al., 2013) for further details on the constraints and conditions.

Another related work is the *orienteering problem* where the objective is to find a path of a fixed length from a single source that visits as many locations as possible (Bansal et al., 2004; Blum et al., 2007; Campbell et al., 2011). The EV touring problem can be regarded as an extension of this problem because the goal is to visit as many cities as possible under a fixed (i.e., full) battery capacity.

Compared with the widely studied routing problems, there is a dearth of research on the optimal *refueling problem* (Khuller et al., 2011; Lin, 2008; Lin et al., 2007; Suzuki, 2008; 2009), where the objective is to minimize the total cost of the fuel used. Lin (2008), Lin et al. (2007) investigated the shortest path problem with optimal refueling policies. They proposed a linear time algorithm for the fixed route version and polynomial time algorithms for other variations. Suzuki (Suzuki, 2008; 2009) developed a more comprehensive model that incorporates many operating costs, and conducted numerical studies. Recently, Khuller et al. (2011) proposed the *gas station problem* where the price of gas may vary at every station, so the owner of a petrol-powered vehicle must decide the amount of gas he/she will purchase (i.e., a fraction of the tank's capacity) at a particular gas station in order to minimize the total cost of gas required. They also study the *tour gas station problem*, where the objective is to find the cheapest tour that can visit a set of vertices and return to the origin, so that the total cost of the gas required is minimized.

Subsequently, Erdoğan and Miller-Hooks presented the *green vehicle routing problem* (Erdoğan and Miller-Hooks, 2012) and Schneider et al. proposed the similar *electric vehicle routing problem* (Schneider et al., 2014). They combined the *vehicle routing problem* with the possibility of filling alternative fuels or charging a vehicle's battery at stations along the routes. Both works provided meta-heuristic algorithms and an analysis of numerical experiments. The setting of the problems is similar to that of the above optimal refueling problem; the only difference is that the objective is to minimize the total distance instead of the total fuel cost. Recently, Adler and Mirchandani (2014) considered routing of many electric vehicles through a network of battery switch stations and developed an algorithm to control battery swap loads across the stations such that the average delay of every vehicle is minimized. In addition, a number of previous studies (Artmeier et al., 2010; Eisner et al., 2011; Kobayashi et al., 2011; Sachenbacher et al., 2011; Siddiqi et al., 2011) discussed energy efficient routing of electric vehicles.

To facilitate the understanding of the difference between our models and the models studied in the literature, we next formally introduce the gas station problem.