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Effect of stochastic transition in the fundamental diagram of traffic flow

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A B S T R A C T

In this work, we propose an alternative stochastic model for the fundamental diagram of traffic flow with minimal number of parameters. Our approach is based on a mesoscopic viewpoint of the traffic system in terms of the dynamics of vehicle speed transitions. A key feature of the present approach lies in its stochastic nature which makes it possible to study not only the flow-concentration relation, namely, the fundamental diagram, but also its uncertainty, namely, the variance of the fundamental diagram—an important characteristic in the observed traffic flow data. It is shown that in the simplified versions of the model consisting of only a few speed states, analytic solutions for both quantities can be obtained, which facilitate the discussion of the corresponding physical content. We also show that the effect of vehicle size can be included into the model by introducing the maximal congestion density k_{max}. By making use of this parameter, the free flow region and congested flow region are naturally divided, and the transition is characterized by the capacity drop at the maximum of the flow-concentration relation. The model parameters are then adjusted to the observed traffic flow on the I-80 Freeway Dataset in the San Francisco area from the NGSIM program, where both the fundamental diagram and its variance are reasonably reproduced. Despite its simplicity, we argue that the current model provides an alternative description for the fundamental diagram and its uncertainty in the study of traffic flow.

1. Introduction

Aside from its complexity and nonlinearity, traffic flow modeling has long attracted the attention of physicists due to the connections to transport theory and hydrodynamics (For reviews, see for example (Hoogendoorn and Bovy, 2001; Kerner, 2009; Maerivoet and De Moor, 2005; Pedersen, 2011; Prigogine and Herman, 1971; Treiber and Kesting, 2012)). Corresponding to the three main scales of observation in physics, traffic flow models can generally be categorized into three classes, namely, microscopic, mesoscopic and macroscopic approaches. The macroscopic models (Costeseque and Lebacque, 2014; Edie, 1960; Greenberg, 1959; Helbing, 1995, 1996; Karlsen, 1995; Kerner and Konhauser, 1993, 1994; Li and Zhang, 2013; Lighthill and Whitham, 1955; Zhang, 2001) describe the traffic flow at a high level of aggregation, where the system is

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treated as a continuous fluid without distinguishing its individual constituent parts. In this approach, the traffic stream is represented in terms of macroscopic quantities such as flow rate, density and speed. Many methods in the conventional hydrodynamics thus can be directly borrowed into the investigation of traffic flow. For instance, one may discuss shock waves (Edie, 1960; Lighthill and Whitham, 1955), the stability of the equation of motion (Kerner and Konhauser, 1993, 1994), or investigate the role of viscosity (Helbing, 1996) analogous to those for real fluid. Mathematically, the problem is thus expressed in terms of a system of partial differential equations. The microscopic approach, on the other hand, deals with the space-time behavior of each individual vehicle as well as their interactions at the most detailed level. In this case, an ordinary differential equation is usually written down for each vehicle. Due to its mathematical complexity, approximation is commonly introduced in order to obtain asymptotic solutions or to make the problem less computationally expensive. The car-following models (Alvarez et al., 1990; Chen et al., 2012; Gazis et al., 1959; Li and Ouyang, 2011; Mu and Yamamoto, 2013; Pipes, 1953; Rathi and Santiago, 1990; Zhang and Kim, 2005), optimal velocity models (Bando et al., 1995; Jiang et al., 2015; Komatsu and Sasa, 1995; Sheu and Wu, 2015; Treiber et al., 2000) and the cellular automaton (Eisenblatter et al., 1998; Nagel and Schreckenberg, 1992a, 1992b; Rickert et al., 1996; Schadschneider and Schreckenberg, 1993) all can be seen as microscopic approaches in this context. For certain cases, such as Greenberg's logarithmic model (Gazis et al., 1959; Greenberg, 1959), the above two approaches were shown to be equivalent in reproducing the fundamental diagram of traffic flow. A mesoscopic model (Nelson, 1995; Paveri-Fontana, 1975; Prigogine and Andrews, 1960; Prigogine and Herman, 1971) seeks compromise between the microscopic and the macroscopic approaches. The model does not attempt to distinguish nor trace individual vehicles, instead, it describes traffic flow in terms of vehicle distribution density as a continuous function of time, spatial coordinates and velocities. The dynamics of the distribution function, following methods of statistical mechanics (Huang, 1987), is usually determined by an integro-differential equation such as the Boltzmann equation. Most mesoscopic models are derived in analogy to gas-kinetic theory. As it is known that hydrodynamics can be obtained by using the Boltzmann equation (Chapman et al., 1991; Grad, 1949; Groot, 1980), the mesoscopic model for traffic flow has also been used to obtain the corresponding macroscopic equations (Helbing, 1995; 1996). These efforts thus provide a sound theoretical foundation for macroscopic models, besides heuristic arguments and lax analogies between traffic flow and ordinary fluids.

One important empirical measurement for a long homogeneous freeway system is the so called "fundamental diagram" of traffic flow. It is plotted in terms of vehicle flow q as a function of vehicle density k:

$$q = q(k). \tag{1}$$

In a macroscopic theory, when the dynamics of the system is determined by an Euler-like or Navier–Stokes-like equation of motion, the fundamental diagram can be derived. Alternatively, one may use the fundamental diagram as an input together with the conservation of vehicle flow and the initial conditions to determine the temporal evolution of the system. Also, the equation of motion of either the microscopic or the mesoscopic model can be employed to calculate the fundamental diagram. The resulting theoretical estimations from any of the above approaches can then be used to compare to the empirical observations which have been accumulated on highways in different countries for nearly 8 decades (see for instance ref. (Daganzo, 2002; Kerner and Rehborn, 1996; Pedersen, 2011)). The following common features are observed in most of the data: (1) Usually the flow-concentration curve is divided into two different regions of lower and higher vehicle density, which correspond to "free" and "congested" flow; (2) The maximum of the flow occurs at the junction between the free and congested region and (3) Congested flow in general presents a broader scattering of the data points on the flow-concentration plane, in comparison to that of the free flow. In other words, the variance of flow for free traffic flow is relatively small, it increases as the vehicle density increases, and eventually the system becomes unstable or chaotic toward the onset of traffic congestion. For this very reason, it is understood by many authors that the transition from free traffic to congestion is a *phase transition*. Most traffic flow models are able to reproduce the main features of the observed fundamental diagram; in particular, traffic congestion is understood to be closely connected to the instability of the equation of motion (Bando et al., 1995; Ben-Naim et al., 1994; Kerner and Konhauser, 1993; Komatsu and Sasa, 1995; Treiber et al., 2000), or to the phase transition of the system (Arnold, 1994; Eisenblatter et al., 1998; Hall, 1987; Kerner, 2009). On the other hand, uncertainty is also observed in the data which can be mostly expressed in terms of the variance of the fundamental diagram. The latter has been an intriguing topic in the recent years (Cassidy, 1998; Castillo and Benitez, 1995; Kerner, 2004; Li et al., 2012; Nelson and Sopasakis, 1998; Treiber and Helbing, 2003). In fact, methodologies involving stochastic modeling have aroused much attention, either from the macroscopic viewpoint (Boel and Mihaylova, 2006; Jabari and Liu, 2012; Sumalee et al., 2011; Wang and Papageorgiou, 2005), from the microscopic aspect such as car-following and cellular automaton models (Dailey and Cathey, 2002; Huang et al., 2001; Nagel and Schreckenberg, 1992a; Schadschneider and Schreckenberg, 1993; Sopasakis and Katsoulakis, 2006; Wagner, 2011), or from other phenomenological approaches (Brilon et al., 2005; Ngoduy, 2011; Wang et al., 2006) such as those introduce uncertainty directly into the fundamental diagram or road capacity.

The present work follows the above line of thought to explore the stochastic nature of traffic flow. First, we will employ a proper mathematical tool to tackle the problem. One notes that a model simple in its mathematical form may not imply the most appropriate interpretation for an elementary physical system. As it is well known, the random motion of particles suspended in a fluid, known as the Brownian motion, is best described by the Wiener process. The latter involves the rules of stochastic calculus since the corresponding equation of motion, a stochastic differential equation (SDE), is typically not differentiable. Secondly, in our approach, one demands the model to be of microscopic/mesoscopic origin, meanwhile it shall not be subjected to special rules tailored for a specific traffic scenario or some certain experimental data, so that the