Transportation Research Part B 83 (2016) 74–90

Contents lists available at ScienceDirect

Transportation Research Part B

journal homepage: www.elsevier.com/locate/trb

Second best toll pricing within the framework of bounded rationality

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ARTICLE INFO

Article history:
Received 22 January 2015
Revised 31 October 2015
Accepted 2 November 2015
Available online 11 December 2015

Keywords:
Bounded rationality (BR)
Toll pricing
Risk-prone
Risk-averse

ABSTRACT

The network design problem is usually formulated as a bi-level program, assuming the user equilibrium is attained in the lower level program. Given boundedly rational route choice behavior, the lower-level program is replaced with the boundedly rational user equilibria (BRUE). The network design problem with boundedly rational route choice behavior is understudied due to non-uniqueness of the BRUE. In this study, thus, we mainly focus on boundedly rational toll pricing (BR-TP) with affine link cost functions. The topological properties of the lower level BRUE set are first explored. As the BRUE solution is generally non-unique, urban planners cannot predict exactly which equilibrium flow pattern the transportation network will operate after a planning strategy is implemented. Due to the risk caused by uncertainty of people's reaction, two extreme scenarios are considered: the traffic flow patterns with either the minimum system travel cost or the maximum, which is the "risk-prone" (BR-TP-RP) or the "risk-averse" (BR-TP-RA) scenario respectively. The upper level BR-TP is to find an optimal toll minimizing the total system travel cost, while the lower level is to find the best or the worst scenario. Accordingly BR-TP can be formulated as either a min–min or a min–max program. Solution existence is discussed based on the topological properties of the BRUE and algorithms are proposed. Two examples are accompanied to illustrate the proposed methodology.

1. Introduction and motivation

1.1. Network design problem

The network design problem (NDP) is to relieve road network congestion by determining how much toll pricing should be charged on congested links, by selecting which links need to be extended, or by considering where new links should be added, given the traffic demand for each origin-destination pair. There have been many studies on transportation network design and interested readers can refer to Yang and Bell (1998) for detailed reviews of the NDP literature.

Among these studies, NDP is usually formulated as a bi-level program: the upper level is the decision made to either execute toll pricing, enhance capacities of the selected links, or add new links to the existing road network, while the lower level is an equilibrium problem, describing how drivers will distribute among the new network topology. Accordingly, the decision variables in the upper level represent whether to charge tolls, to widen a link, or to build a new road, and can be either continuous

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http://dx.doi.org/10.1016/j.trb.2015.11.002

or discrete. The lower level problem is the link or path flow distribution based on some traffic assignment principle. When Wardrop's first principle (Wardrop, 1952) is adopted, the user equilibrium (UE) is obtained in the lower level problem.

The above bi-level NDP can be also regarded as a Stackelberg game (i.e., leader-follower game Yang and Bell (1998)). The leader is the decision-maker managing the upper level variables, while individual drivers are followers who play the non-cooperative Nash game on the fixed topology of the road network. In other words, provided the upper level decision, each driver will response accordingly by playing a Nash game and choose her own route in response to the upper level decision.

1.2. Non-uniqueness of the lower level program

If the lower level problem has only one equilibrium (i.e., each driver has a single action strategy in response to a network change plan), planners can make an exact prediction of how drivers will react to a certain transportation plan. However, if there exist multiple equilibria in the lower level problem (i.e., each driver has multiple action strategies), planners do not know exactly how drivers will behave, then solving a standard bi-level program may not achieve the expected goal of improving efficiency of the transportation system. Because the bi-level program "implicitly assumes that by applying the obtained toll, the resulting UE flow pattern is exactly what is predicted by the model, and thus the desired system objective can be achieved" (Ban et al., 2009). Ban et al. (2009) also proposed two concepts: the "predicted" and the "realized" equilibrium, to further explain consequences resulting from implementing a toll obtained from the bi-level program when the lower level problem does not have a unique solution. The former represents the equilibrium obtained from solving the bi-level program, while the latter means the actual equilibrium where the system runs after a network plan is implemented. As drivers play a non-cooperative Nash game in the lower level, there is no way to control which routes they can take. Therefore, the "realized" equilibrium usually deviates from the "predicted" one.

Due to the gap between the "predicted" and the "realized" equilibria, planners face risk or uncertainty while making network plans. There are three attitudes towards risk encountered in the NDP (Ban et al., 2009): risk-prone, risk-neutral and risk-averse. "Risk prone" assumes that planners are optimistic and believe the realized equilibrium will be attained at the most efficient flow pattern, i.e., the one with the lowest system travel time; then they try to find a toll which can improve system efficiency at this equilibrium. In contrast, "risk averse" represents the attitude that planners try to find a toll which can improve system efficiency by assuming the realized equilibrium is attained at the least efficient flow pattern, i.e., the one with the largest system travel time. Planners are "risk-neutral" when they assume each flow pattern has certain probability to occur and they aim to find a toll which can improve system efficiency at the expected equilibrium (Ban et al., 2013).

1.3. NDP with boundedly rational user equilibria in the lower level

Regarding the lower level traffic assignment problem, most NDP literature assumes drivers are perfectly rational (PR) and its resultant equilibrium follows Wardrop's first principle (e.g., Leblanc 1975; Magnanti and Wong 1984; Yang and Bell 1998). However, many empirical studies using simulation experiments (Nakayama et al., 2001), stated preference surveys (Avineri and Prashker, 2004), and GPS vehicle trajectory data (Morikawa et al., 2005; Zhu, 2011) showed that in reality, this assumption is too restrictive and drivers do not always choose the shortest paths. Thus, many other behavioral models have been developed to relax the PR assumption for the traffic assignment problem (see Xu et al. (2011) for the classification of these models).

One way to relax the PR assumption is to assume people are boundedly rational (BR) in the route-choice process. Under the BR route choice paradigm, each traveller has a set of paths to choose instead of travelling only on the shortest paths. The paths which are within a threshold from the shortest path cost can be taken, where this threshold is called "indifference band." There have been many empirical and experimental studies to establish validity of bounded rationality and interested readers can refer to the associated literature (Di, 2014; Di et al., 2014a; Di and Liu, 2015; Di et al., 2015a, 2015b; Hu and Mahmassani, 1997; Jan et al., 2000; Mahmassani and Chang, 1987; Morikawa et al., 2005; Ramming, 2001).

Within the BR framework, the equilibrium obtained from the lower level traffic assignment problem is not UE any more; instead, it is boundedly rational user equilibria (BRUE). Usually the BRUE is not unique and the set consisting of all BRUE flow patterns is called the 'BRUE flow set'. This set has been proven to be non-convex (Di et al., 2013; Lou et al., 2010). Therefore it is quite challenging to solve the NDP with boundedly rational user equilibria (BR-NDP). Within the 'risk averse' scheme, Lou et al. (2010) minimized the 'worst case' scenario (i.e., assuming the network runs least efficiently in terms of the total system travel time) among all BRUE solutions.

1.4. Contribution of this paper

Because the network design problem with bounded rationality behavioral assumption (BR-NDP) is a challenging problem and there are not many established studies, we will mainly focus on the continuous version of the NDP: the second best toll pricing with boundedly rational user equilibria (BR-TP), given the affine link cost functions. The second best toll pricing is to determine an optimal toll vector for a subset of links so that the transportation network can run more efficiently (Ban et al., 2009).

Some existing literature proposed methodologies of solving BRUE (Di et al., 2013; Lou et al., 2010), however, there does not exist any study on the topological properties of BRUE sets, which can facilitate applications of BR-NDP. This paper serves as the first to explore topology of BRUE sets.