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# A two-stage stochastic optimization model for the transfer activity choice in metro networks

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ABSTRACT

This research focuses on finding the best transfer schemes in metro networks. Using sample-based time-invariant link travel times to capture the uncertainty of a realistic network, a two-stage stochastic integer programming model with the minimized expected travel time and penalty value incurred by transfer activities is formulated. The first stage aims to find a sequence of potential transfer nodes (stations) that can compose a feasible path from origins to destinations in the transfer activity network, and the second stage provides the least time paths passing by the generated transfer stations in the first stage for evaluating the given transfer schemes and then outputs the best routing information. To solve our proposed model, an efficient hybrid algorithm, in which the label correcting algorithm is embedded into a branch and bound searching framework, is presented to find the optimal solutions of the considered problem. Finally, the numerical experiments are implemented in different scales of metro networks. The computational results demonstrate the effectiveness and performance of the proposed approaches even for the large-scale Beijing metro network.



## 1. Introduction

### 1.1. Motivations

To meet the daily travel demands of passengers, a large number of metro lines have been put into operations or being under construction in a lot of cities, due to the characteristics of punctuality, large transportation capacity, high-efficiency, etc. For instance, in Beijing city, the total length of the current and in-construction metro lines have been over 500 km up to 2015, causing the drastic expansion of exiting metro networks. Typically, in a large and complex urban rail transit network, optimizing travel choice behavior is usually regarded as a key issue concerned by travelers in planning and specifying their trips. An effective routing strategy in the network can not only provide efficient guidance information for travelers, but also lead to the potential utilization optimization of existing resources and infrastructure. This might involve in selecting favorite transfer schemes (or paths) before the departure or en route when one's origin and destination locate on different metro lines.

A survey in Shanghai city shows that, about fifty percent of travel time is usually occupied by the transfer process when one travels in the metro system[1]. Also in Beijing metro network, more than forty percent of passengers need to experience the transfer activities during their trips, in which most transfer processes are fairly complicated due to the complex infrastructure of transfer

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[1] http://shanghai.xinmin.cn/msrx/2014/12/18/26243754.html

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stations and large passenger flows (there are more than ten million passengers in the metro system on each weekday)(Wang, 2014)[2]. Consequently, more than ten transfer stations have to provide services for daily passenger volume over 120 thousand where most transfer activities occur during the peak hours[3]. Due to the importance of transfer activity in path finding, how to choose favorite and robust transfer strategies in the current situations turns out to be a theoretically significant issue for the real-world applications of traffic participants.

In the literature, the path finding in a network always focuses on producing the least travel time route. However, note that the involved travel time not only includes the travel time in a train, but also consists of passenger's walking time and waiting time at platforms incurred by transfer activities from one metro line to another. Thus, the travel time in a large-scale metro network is inevitably associated with stochastic features due to the disturbances of passenger flows, layout of transfer stations, train delay propagation and other indeterminate factors. Combining with these uncertain information, we intend to generate the robust transfer scheme through a two-stage decision making process, in which both a priori optimization and adaptive decision are integrated in the solution framework. This research will explicitly address the above mentioned problems.

### *1.2. Literature review*

Finding the best transfer scheme in a metro network, which is a common behavior when passengers take the sub-way system or pre-plan their trips, can be included in the framework of stochastic shortest path problems. Generally, the classical shortest path problem often adopts deterministic arc lengths or arc travel times to represent network characteristics. A 0-1 integer programming model, which is efficiently solved by the label setting/correcting algorithms proposed by Bellman (1958), Dijkstra (1959), Dreyfus (1969), can be used to formulate the shortest path choice process. Note that, however, the link travel time in real-world transportation networks is often stochastic and uncertain. To handle this problem within uncertain decision environments, a variety of modelling methods and solution algorithms have been proposed by many researchers in the literature based on the stochastic programming methods (For more details of stochastic programming, please see Ben-Tal and Nemirovski, 2000; Birge and Louveaux, 1997; Chen et al., 2007; Goel and Grossmann, 2006; Heitsch and Romisch, 2003; Kall and Mayer, 2005). For instance, Frank (1969), Mirchandani (1976) and Sigal et al. (1980) explored the probability distribution of the shortest path lengths in stochastic networks. Mirchandani and Soroush (1986); Murthy and Sarkar (1996); Loui (1983) considered the different types of cost functions to determine optimal paths in stochastic networks from the perspective of decision-maker's utility.

To the best of our knowledge, the majority of the existing literature mainly focus on two evaluation indexes in path choice, namely the expected utility or non-expected utility (see Chen and Nie, 2015; Fan and Nie, 2006; Fu and Rilett, 1998; Huang and Gao, 2012; Nielsen, 2003; Miller-Hooks, 2001; Nie and Wu, 2009; Waller and Ziliaskopoulos, 2002; Yang et al., 2013; Yang and Zhou, 2014; Fan et al., 2005; Wang et al., 2016; Wu, 2015), in which the most commonly used objective is to find the least expected travel time. Hall (1986) first illustrated that standard shortest path algorithms can not be used to find the least expected travel time path in a network whose travel times are both random and time-dependent, and then proposed an effective method to solve this problem. Fu and Rilett (1998) defined link travel times as continuous time stochastic process and developed a general probability-based formula to calculate the mean and variance of the travel time for a given path in a dynamic and stochastic network. Miller-Hooks and Mahmassani (2000) proposed two specialized modified label correcting algorithms, namely the expected value algorithm and the expected lower bound algorithm, to determine least expected time paths in stochastic, time-varying networks. In addition, Miller-Hooks (2001) presented an efficient label-setting algorithm to determine adaptive least expected time hyperpaths in stochastic networks. Yang and Miller-Hooks (2004) considered additional delay due to signal operations in signalized traffic networks, to obtain the least expected travel time hyperpaths in signalized stochastic networks. Gao and Chabini (2006) raised a more general explanation to the least expected travel time path, which viewed that the least expected time path problem was an optimal routing policy problem with additional side constraints. They established a framework to the optimal routing policy problems in stochastic networks and gave an exact algorithm.

### *1.3. The focus and proposed methods*

Here, it is worth noting that, the main focus of this research is not on the a priori route generation but on the robust transfer activity choice in a metro network. Compared to the road networks in large cities, the characteristics of metro networks are much different since the majority of operations of trains are required to follow a pre-designed and well-organized schedule. Nevertheless, due to the complexity of the operational environments, there are still significant stochastic characteristics associated with the practical metro lines. For instance, owing to the large amount of passenger flow in peak hours (this is the situation in Beijing metro system), the actual dwelling time of trains on busy platforms always fluctuates around some pre-scheduled values in the process of boarding/unloading passengers. Thus, from the perspective of passengers, the total travel time in a train is usually uncertain if one takes the train from his/her origin to destination. Moreover, if one needs to transfer at some stations, the required transfer time will be fully dependent on a series of factors such as current passenger volume, station inner structure, etc. Accordingly, the transfer time will also be stochastic in most circumstances. In addition, passenger flow control in peak hours is a frequently-used method in transfer stations to manage the states of the railway traffic and transportation, causing the potential

[2] http://news.xinhuanet.com/local/2015-04/17/c_127701848.html
[3] http://www.chinanews.com/life/2015/01-16/6975222.shtml