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Improved bush-based methods for network contraction

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A R T I C L E I N F O

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A B S T R A C T

Calculating equilibrium sensitivity on a bush can be done very efficiently, and serve as the basis for a network contraction procedure. The contracted network (a simplified network with a few nodes and links) approximates the behavior of the full network but with less complexity. The network contraction method can be advantageous in network design applications where many equilibrium problems must be solved for different design scenarios. The network contraction procedure can also be used to increase the accuracy of subnetwork analysis. This method requires calculating travel time derivatives between two nodes, with respect to the demand between them, assuming that the flow distributes in a way that equilibrium is maintained. Previous research describes two methods for calculating these derivatives. This paper presents a third method, which is simpler, faster, and just as accurate. The method presented in this paper reformulates the linear system of equations defining these sensitivities as the solution to a convex programming problem, which can be solved by making minor modifications to static user equilibrium algorithms. In addition, the model is extended to capture the interactions between the path travel times and network flows, and a heuristic is proposed to compute these interactions. The accuracy and complexity of the proposed methodology are evaluated using the network of Barcelona, Spain. Further, numerical experiments on the Austin, Texas regional network validate its performance for subnetwork analysis applications.

1. Introduction

Static traffic assignment remains the most common network equilibrium model in practice: its favorable mathematical properties are well-known, practitioners are experienced at calibrating and interpreting the results, and it can be solved quickly. In particular, algorithms based on bushes — a concept dating to Dial (1970), and first applied to the static equilibrium problem in Dial (1999a) — exploit the acyclicity of the equilibrium flows to solve for equilibrium rapidly.

An important consequence of Beckmann et al.'s formulation (1956) is that a meaningful sensitivity analysis can be undertaken. In the context of network equilibria, sensitivity analysis refers to determining a functional relationship between the travel times and demands without re-solving the network equilibrium problem. This functional relationship can be used to represent the network by connecting each origin-destination (OD) pair with a single artificial link and removing all the intermediate nodes and links.

The main reason to develop such network contraction techniques is reducing the computational burden of solving many network equilibrium problems (Friesz, 1985). Still this computational motivation seems to be valid. As an example, consider a network design problem which is formulated as a bi-level problem. The master problem deals with computing some design

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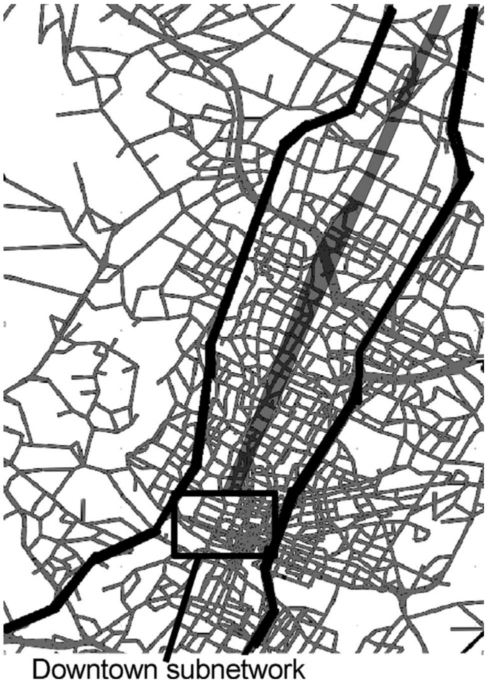

Fig. 1. Portion of regional network with equidistant region shaded. Route choice from this region to downtown is sensitive to changes in the subnetwork regardless of distance between the origin and downtown.

parameters, and in the equilibrium subproblem, the travelers modify their route choice in response to design parameters set in the master problem. Such problems require solving hundreds or thousands of the network equilibrium problem as a subproblem which can be computationally expensive even for modern algorithms such as Algorithm B (Dial, 2006), TAPAS (Bar-Gera, 2010), or LUCE (Gentile, 2014).

As another application, consider the case of studying a number of alternatives, such as a new signal timing plan or converting a one-way local street to two ways, in a region of interest. The impacts of these policies are expected to be local, and solving the whole regional network may be unnecessary. Subnetwork analysis is commonly used to avoid incurring the computational burden of regional modeling. In practice, subnetwork modeling usually involves extracting a small component of a regional network, allowing the boundary nodes of the subnetwork to serve as origins and destinations, and estimating the subnetwork trip table from an equilibrium solution on the regional network. Xie et al. (2010) use entropy-maximization to identify these trips between the boundary points of the subnetwork. Effectively, this forms a “fixed” boundary condition which neglects diversion effects due to changes in the subnetwork.

Boyles (2012) adopts a different approach, using bush-based sensitivity analysis to simplify the regional network, rather than delete it entirely. In this way, diverting flows can be approximated while still retaining most of the computational advantage of subnetwork modeling; the boundary is less rigidly enforced. The chief advantage of this procedure is that it captures diversion in a behavioral manner, based on drivers choosing routes to minimize their travel time. This contrasts with the fixed-boundary approach, where a common question is “how large does the subnetwork need to be to capture diversion?” While natural, this question is a bit of a red herring. Consider a downtown area served by two roughly parallel freeways (Fig. 1). Even at large distances from the downtown area, there are drivers whose origins are roughly equidistant from these two freeways. These drivers’ choice between these freeways depends on travel patterns downtown, regardless of how distant their origin is: the fundamental issue is modeling route choice, not simply capturing a large enough area. By integrating one model into another, “smoothing” the boundary, much faster convergence and greater accuracy can be seen without needing a large subnetwork.

Creating such a model requires estimation of a number of sensitivity parameters. Boyles (2012) provided two methods to calculate these parameters: one is reminiscent of resistive circuit analysis, and only applies when the equilibrium bushes are planar. The other is based on iterative solution of a linear system, exploiting the underlying network structure to avoid inverting any large matrices. Both of these methods, however, have undesirable aspects. In modern regional networks, planar bushes are rare because freeway interchanges and overpasses are modeled in detail, rather than representing the entire interchange with a single node (as was done in past decades to reduce the number of nodes and arcs). For example, in the Chicago regional network (Bar-Gera, 2001), none of the 1790 origin bushes are planar at equilibrium. While the second method is applicable in general networks, it requires careful implementation to avoid numerical instabilities due to repeated matrix reinversion.

Also, these previous methods assume that the travel time between the origin and destination nodes of each OD pair is a function of its own demand, and independent of other OD demands. In case of a congested network, this assumption may degrade