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Finding optimal solutions for vehicle routing problem with pickup and delivery services with time windows: A dynamic programming approach based on state–space–time network representations

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ABSTRACT

Optimization of on-demand transportation systems and ride-sharing services involves solving a class of complex vehicle routing problems with pickup and delivery with time windows (VRPPDTW). This paper first proposes a new time-discretized multi-commodity network flow model for the VRPPDTW based on the integration of vehicles' carrying states within space–time transportation networks, so as to allow a joint optimization of passenger-to-vehicle assignment and turn-by-turn routing in congested transportation networks. Our three-dimensional state–space–time network construct is able to comprehensively enumerate possible transportation states at any given time along vehicle space–time paths, and further allows a forward dynamic programming solution algorithm to solve the single vehicle VRPPDTW problem. By utilizing a Lagrangian relaxation approach, the primal multi-vehicle routing problem is decomposed to a sequence of single vehicle routing subproblems, with Lagrangian multipliers for individual passengers' requests being updated by sub-gradient-based algorithms. We further discuss a number of search space reduction strategies and test our algorithms, implemented through a specialized program in C++, on medium-scale and large-scale transportation networks, namely the Chicago sketch and Phoenix regional networks.

1. Introduction

As population and personal travel activities continue to increase, traffic congestion has remained as one of the major concerns for transportation system agencies with tight resource constraints. The next generation of transportation system initiatives aims to integrate various demand management strategies and traffic control measures to actively achieve mobility, environment, and sustainability goals. A number of approaches hold promises of reducing the undesirable effects of traffic congestion due to driving-alone trips, to name a few, demand-responsive transit services, dynamic ride-sharing, and intermodal traffic corridor management.

The optimized and coordinated ride-sharing services provided by transportation network companies (TNC) can efficiently utilize limited vehicle and driver resources while satisfying time-sensitive origin-to-destination transportation service requests. In a city with numerous travelers with different purposes, each traveler has his own traveling schedule. Instead of

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using his own car, the traveler can (by the aid of ride-sharing) bid and call a car just a few minutes before leaving his origin, or preschedule a car a day prior to his departure. The on-demand transportation system provides a traveler with a short waiting time even if he resides in a high-demand area. Currently, several real-time ride-sharing or, more precisely, app-based transportation network and taxi companies, such as Uber and Lyft are serving passengers in many metropolitan areas. In the long run, a fully automated and optimized ride-sharing approach is expected to handle very complex transportation supply-to-demand assignment tasks and offer a long list of benefits for transportation road users and TNC operators. These benefits might include reducing driver stress and driving cost, improving mobility for non-drivers, increasing safety and fuel efficiency, and decreasing road congestion as well as reducing overall societal energy use and pollution.

The ride-sharing problem can be mathematically modeled by one of the well-known optimization problems which is the vehicle routing problem with pickup and delivery (VRPPD). In this paper, in order to improve the solution quality and computational efficiency of on-demand transportation systems and dynamic ride-sharing services, especially for large-scale real-world transportation networks, we propose a new mathematical programming model for the vehicle routing problem with pickup and delivery with time windows (VRPPDTW) that can fully recognize time-dependent link travel time caused by traffic congestion at different times of day. Based on the Lagrangian relaxation solution framework, we further present a holistic optimization approach for matching passengers' requests to transportation service providers, synchronizing transportation vehicle routing, and determining request pricing (e.g. through Lagrangian multipliers) for balancing transportation demand satisfaction and resource needs on urban networks.

2. Literature review and research motivations

The vehicle routing problem with pickup and delivery with time windows (VRPPDTW) or simply, pickup and delivery problem with time windows (PDPTW), is a generalized version of the vehicle routing problem with time windows (VRPTW), in which each transportation request is a combination of pickup at the origin node and drop-off at the destination node (Desaulniers et al., 2002). The PDPTW under consideration in this paper contains all constraints in the VRPTW plus added constraints in which either pickup or delivery has given time windows, and each request must be served by a single vehicle. The PDPTW may be observed as the dial-a-ride problem in the literature as well. Since the VRPTW is an NP-hard problem, the PDPTW is also NP-hard (Baldacci et al., 2011).

Several applications of the VRPPDTW have been reported in road, maritime, and air transportation environments, to name a few, Fisher et al. (1982), Bell et al. (1983), Savelsbergh and Sol (1998), Wang and Regan (2002), and Zachariadis et al. (2015) in road cargo routing and scheduling; Psaraftis et al. (1985), Fisher and Rosenwein (1989), and Christiansen (1999) in sea cargo routing and scheduling; and Solanki and Southworth (1991), Solomon et al. (1992), Rappoport et al. (1992), and Rappoport et al. (1994) in air cargo routing and scheduling. Further applications of the VRPPDTW can be found in transportation of elderly or handicapped people (Jaw et al., 1986; Alfa, 1986; Ioachim et al., 1995; and Toth and Vigo, 1997), school bus routing and scheduling (Swersey and Ballard, 1983; and Bramel and Simchi-Levi, 1995), and ride-sharing (Hosni et al., 2014; and Wang et al., 2015). Recently, Furuhata et al. (2013) offers an excellent review and provides a systematic classification of emerging ride-sharing systems.

Although clustering algorithms (Cullen et al., 1981; Bodin and Sexton, 1986; Dumas et al., 1989; Desrosiers et al., 1991; and Ioachim et al., 1995), meta-heuristics (Gendreau et al., 1998; Toth and Vigo, 1997; and Paquette et al., 2013), neural networks (Shen et al., 1995), and some heuristics such as double-horizon based heuristics (Mitrovic-Minic et al., 2004) and regret insertion heuristics (Diana and Dessouky, 2004) have been shown to be efficient in solving a particular size of PDPTW, in general, finding the exact solution via optimization approaches has still remained theoretically and computationally challenging. Focusing on the PDPTW for a single vehicle, Psaraftis (1980) presented an exact backward dynamic programming (DP) solution algorithm to minimize a weighted combination of the total service time and the total waiting time for all customers with $O(n^2 3^n)$ complexity. Psaraftis (1983) further modified the algorithm to a forward DP approach. Sexton and Bodin (1985a, 1985b) decomposed the single vehicle PDPTW to a routing problem and a scheduling sub-problem, and then they applied Benders' decomposition for both master problem and sub-problem, independently. Based on a static network flow formulation, Desrosiers et al. (1986) proposed a forward DP algorithm for the single-vehicle PDPTW with the objective function of minimizing the total traveled distance to serve all customers. After presenting our proposed model in the later section, we will conduct a more systematical comparison between our proposed state–space–time DP framework and the classical work by Psaraftis (1983) and Desrosiers et al. (1986).

There are a number of studies focusing on the multi-vehicle pickup and delivery problem with time windows. Dumas et al. (1991) proposed an exact algorithm to the multiple vehicle PDPTW with multiple depots, where the objective is to minimize the total travel cost with capacity, time window, precedence and coupling constraints. They applied a column generation scheme with a shortest path sub-problem to solve the PDPTW, with tight vehicle capacity constraints, and a small size of requests per route. Ruland (1995) and Ruland and Rodin (1997) proposed a polyhedral approach for the vehicle routing problem with pickup and delivery. Savelsbergh and Sol (1998) proposed an algorithm for the multiple vehicle PDPTW with multiple depots to minimize the number of vehicles needed to serve all transportation requests as the primary objective function, and minimizing the total distance traveled as the secondary objective function. Their algorithm moves toward the optimal solution after solving the pricing sub-problem using heuristics. They applied their algorithm for a set of randomly generated instances. In a two-index formulation proposed by Lu and Dessouky (2004), a branch-and-cut algorithm was able to solve problem instances. Cordeau (2006) proposed a branch-and-cut algorithm based on a three-index