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Two-phase stochastic program for transit network design under demand uncertainty

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ABSTRACT

This paper develops a reliability-based formulation for rapid transit network design under demand uncertainty. We use the notion of service reliability to confine the stochastic demand into a bounded uncertainty set that the rapid transit network is designed to cover. To evaluate the outcome of the service reliability chosen, flexible services are introduced to carry the demand overflow that exceeds the capacity of the rapid transit network such designed. A two-phase stochastic program is formulated, in which the transit line alignments and frequencies are determined in phase 1 for a specified level of service reliability; whereas in phase 2, flexible services are determined depending on the demand realization to capture the cost of demand overflow. Then the service reliability is optimized to minimize the combined rapid transit network cost obtained in phase 1, and the flexible services cost and passenger cost obtained in phase 2. The transit line alignments and passenger flows are studied under the principles of system optimal (SO) and user equilibrium (UE). We then develop a two-phase solution algorithm that combines the gradient method and neighborhood search and apply it to a series of networks. The results demonstrate the advantages of utilizing the two-phase formulation to determine the service reliability as compared with the traditional robust formulation that pre-specifies a robustness level.

1. Introduction

Rapid transit systems, such as rapid bus, metro, light rail, are a lifeblood of cities. They enhance mobility and energy efficiency as well as mitigate roadway congestion. The Transit Network Design Problem (TNDP) is to decide the station locations, route alignments and frequencies of rapid transit lines (RTL) to serve the travel demands between specific origin-destination (OD) pairs (Fan and Machemehl, 2006). Transit services often run on fixed schedules which are published in advance. The schedules are not only crucial to achieve cost minimization in managing the fleet size, vehicle and crew scheduling, but also useful to commuters for departure time choices. Due to the stochastic nature of travel demand, it may happen that a heavy demand shows up on one day but a light demand on the other. Indeed, the optimal service network could be altered substantially by incorporating uncertain demand (Lium and Crainic, 2009). Developing a schedule that accommodates the uncertain demand variation in each day with a satisfactory level of comfort has thus attracted a lot of interest. Wang and Meng (2012) studied robust schedule design for liner shipping services considering the uncertainty in container handling time, demand, etc. Apart from the demand side, the uncertainty may arise from the supply side, such as link failure. De-Los-Santos et al. (2012) evaluated the robustness of rail transit network under two scenarios, with or without bridging

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interruptions. Laporte et al. (2010) employed a game theoretic method for railway transit network design considering link failure and competing modes. In this paper, we consider the uncertainty arising from the demand side.

The TNDP is generally formulated as a multi-objective optimization problem considering different objectives, such as (1) maximizing service coverage under budget constraint, (2) minimizing construction and operating costs while maintaining passenger travel time to a satisfactory level (Samanta and Jha, 2011). Route coverage and direct service provision are key elements in determining passengers' mode choices (Hensher and Li, 2012); whereas construction and operating costs determine the economic feasibility or profitability of the system, and thus are major concerns to the operators. The current literature can thus be classified into two categories: one focuses on determining the rapid transit line alignment and station locations to maximize coverage and/or to minimize passenger cost (Yu et al., 2012; Laporte et al., 2005); the other aims at selecting the optimal station locations as well as frequencies with a fixed network topology to minimize the sum of user and operator costs (Wan and Lo, 2003, 2009; Laporte et al., 2007; Bruno et al., 1998). Specifically, Yu et al. (2012) developed an ant colony optimization model to maximize the demand density along a route, where transfers were specifically addressed as an important factor influencing passenger mode choices. Laporte et al. (2005) optimized the location of a single rapid transit line to maximize trip coverage. TNDP in the presence of competing modes, such as private car, is regarded as an extension of current studies. The interaction between the transit network to be constructed and the existing road network further adds complexity to the problem. Wan and Lo (2003, 2009) formulated the multi-modal network design problem. Recently, Perea et al. (2014) proposed a mixed integer non-linear programming model to add a new railway station and a new road junction on a road-rail network in the presence of modal competition.

While the deterministic TNDP has been explored extensively, few studies have investigated the issue of demand uncertainty associated with this problem. Typically there are two methods to address stochastic demand: one is stochastic optimization which aims at minimizing the expected cost by assuming that the demand follows a certain probability distribution; the other is robust optimization which aims at minimizing the cost associated with the worst case scenario (An and Lo, 2015). Robust optimization requires less information on the stochastic demand than stochastic optimization. It assumes that the demand distribution is only partially known, such as first and second moments, and belongs to a given family of probability distribution. One may refer to Gabrel et al. (2014) for a review of recent developments in robust optimization. Stochastic programming typically draws upon a certain sampling method and adopts the sample average to approximate the cost expectation, which can be expressed as a large-size mixed integer linear programming (MILP). It can be solved by commercial software such as CPLEX or by the L-shaped method (Slyke and Wets, 1969; Birge and Louveaux, 1988). Owning to the formidable computational work by exact methods, many studies resort to heuristic algorithms for solving large-size problems, such as neighborhood search methods, genetic algorithms and hybrid search methods (Guihaire and Hao, 2008). Robust optimization focuses on the worst case scenario, and thus is of a similar level of complexity as its deterministic counterpart. The downside is that its solutions may be conservative. Some studies thus turn to redefining the uncertainty set such that all possible realizations within the set are considered, whereas those outside are ignored. In the end, it is important to decide the size of the uncertainty set over which the worst scenario is generated, which ideally should strike a proper balance between the resultant system cost and the level of robustness achieved (Ben-Tal and Nemirovski, 1999; Bertsimas and Sim, 2004). As the size of the uncertainty set is decisive to the solution quality, choosing it inappropriately may, on one hand, fall short of the ability to hedge against uncertainty or, on the other, produce an overly conservative solution. Developing a method to rationally determine the size of the uncertainty set, which would relieve the dependency on expert opinion, is non-trivial. In response to this question, some studies use the cost variance across scenarios as a measure of solution robustness and incorporate it into the objective function. This approach allows an explicit mean-variance tradeoff. List et al. (2003) examined the bus fleet sizing problem via a robust optimization model by incorporating the mean-variance tradeoff in the objective function under demand and vehicle productivity uncertainty. Yan et al. (2012) developed a robust optimization model for scheduling of a fixed bus route aiming at minimizing the sum of the random schedule deviations. These studies typically focus on bus travel time reliability. Both the mean and variance of travel time are important attributes to measure system performance; hence the objective of minimizing a combination of the mean and variance of travel time is sensible. In this study, however, we aim at providing transit services to address demand uncertainty; cost variance, while interesting to know, does not quite capture the objective of the operator to minimize the expected cost.

Robust and stochastic optimizations are two complementary ways to deal with uncertainty, each having its own advantages and disadvantages. In spite of their differences, researchers have made renewed efforts to bridge these two approaches. Bertsimas and Goyal (2010) quantified the performance of robust optimization with different distributions. Chen et al. (2007) investigated stochastic programming from the perspective of robust optimization. In this paper, we provide a stochastic perspective on robust optimization. Namely, instead of fixing the size of the uncertainty set a priori, when more information about the underlying distribution is available, can we limit the size of the uncertainty set in a rational manner so as to strike a proper balance between robustness or reliability of the system and its resultant cost? Such an analysis will be helpful in identifying the value of collecting information about the underlying probability distribution.

In this paper, we develop another way to evaluate the outcome of the uncertainty set in TNDP. Recently, we formulated the TNDP under demand uncertainty for system optimal flows by considering the combination of two services types, (i) rapid transit lines (RTL) or regular services and (ii) demand responsive or flexible services. Defining the notion of service reliability (SR), we proposed a two-phase model to separate the otherwise intertwined decisions over the deployment of these two service types (Lo et al., 2013; An and Lo, 2014a, 2014b). The RTL are designed to cover the stochastic demand up to a certain specified SR. The RTL operate on dedicated right-of-ways or tracks with fixed schedules, whereas flexible services