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An ordinary differential equation formulation of the bottleneck model with user heterogeneity

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ABSTRACT

Considering heterogeneous values of time and schedule delay early, in this paper we develop an ordinary differential equation formulation of the bottleneck model without allowance of arriving late. We show that in no-toll equilibrium, the generalized travel cost increases with departure time as the ratio of value of schedule delay to value of time increases with these two values, and commuters with higher values would experience higher generalized costs. We then derive the first-best toll and analyses its efficiency and distributional impacts. We obtain the sufficient Pareto-improving condition for the first-best scheme without imposing specific functional forms on heterogeneity. It is a Pareto improvement when the gap between values of time and schedule delay early increases as the ratio of value of schedule delay early to value of time increases. The proposed approach is applied to deal with such user heterogeneity that both the values of time and schedule delay early follow uniform distributions. In this case, the Pareto-improving condition is not only sufficient but also necessary. The individual gain from tolling is strictly monotonically increasing with respect to values of time and schedule delay early. Outside this condition, the first-best toll scheme makes users with relatively higher values of time and schedule delay early better off and those with relatively lower values worse off. And, users with higher values of time and schedule delay early could gain more or lose less from tolling. In contrasts with previous literature, our approach is robust in the sense that it can deal with not only discrete but also continuous user heterogeneity.

1. Introduction

A pure bottleneck model with endogenous departure times for commuters was originally presented by Vickrey (1969). In this model, each commuter is confronted with a trade-off between the anticipated costs of travel time and schedule delay (incurred when she/he cannot arrive at the destination at a desired time) because of the bottleneck's finite capacity. The travel cost experienced by a commuter is determined by his or her departure time from home. Every commuter minimizes his/her cost by choosing the departure time, which leads to a cost equilibrium on all commuters. The bottleneck model has been refined subsequently by other scholars (see for example, Hendrickson and Kocur, 1981; Smith, 1984; Daganzo, 1985; Arnott et al., 1990; Yang and Huang, 1997; Pang et al., 2012) and extended along several directions (see for a review, Arnott et al., 1998).

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These studies provide a better understanding of the traveling patterns, congestion externality, and economic means to reduce or eliminate congestion. However, most of these studies have a strong assumption that travelers are homogeneous, i.e., all commuters have the same preference for arriving early or late and have the identical value of time. This is, of course, an unrealistic assumption (Small et al., 2005), because it can significantly influence the results, particularly travel patterns resulting from user equilibrium and also the effects of congestion pricing. Hence, user heterogeneity in the valuation of travel time and schedule delay has attracted much attention since being raised (Small, 1982; Cohen, 1987; Arnott et al., 1994; Huang, 2000; Lindsey, 2004; Verhoef and Small, 2004; Small et al., 2005; Ramadurai et al., 2010; Xiao et al., 2010; van den Berg and Verhoef, 2011a; Liu and Nie, 2011; Hall, 2013).

User heterogeneity is often assumed to be discrete, i.e., all commuters are divided into several number of groups and commuters of one group are assumed to have the same preference for arriving early/late and have the identical value of time. Vickrey (1973) studied a discrete case by introducing "proportional heterogeneity" to the bottleneck model. The proportional heterogeneity means that, although values of time and schedule delay vary across all commuters, the ratio of value of time to value of schedule delay is assumed to be the same for all. Vickrey (1973) found that in no-toll equilibrium, the arrival order is undetermined, but the first-best tolling produces an arrival order in terms of the value of schedule delay. Hence, tolling not only removes queuing, but also lowers scheduling cost, by making the arrival order more efficient. This states that tolling is a strict Pareto improvement as all users gain from this scheme. In fact, tolling is more beneficial under proportional heterogeneity than under homogeneity. Cohen (1987) considered two typical groups of commuters, the low-income commuters who have low value of time but rigid work schedule and the high-income commuters who value their time higher and have more flexible work schedule. Arnott et al. (1988), Arnott et al. (1994) generalized the analyses by Cohen (1987) by incorporating other dimensions of user heterogeneity. Furthermore, de Palma and Lindsey (2002) addressed a different case, in which the heterogeneity is in the value of time, and the values of schedule delay are identical. With fixed demand, all users lose from the first-best tolling—except the users with the very highest value of time, who are unaffected. Recently, Liu et al. (2015) proposed a novel semi-analytical approach for solving the dynamic user equilibrium of a bottleneck model. In their study, the ratio of value of time to value of schedule delay can vary across different groups.

In 1987, Newell dealt with more general heterogeneity that commuters continuously differ in work start time and values of time and schedule delay (Newell, 1987). Using graphic demonstration (e.g., tangent line), he addressed the morning commute problem with and without lateness, respectively. In recent years, a few studies concern continuous values of time and schedule delay. van den Berg and Verhoef (2011a) analyzed the effects of tolling with continuous heterogeneity in the value of time and price-sensitive demand. van den Berg and Verhoef (2011b) investigated the efficiency and distributional impacts of tolling through studying a bottleneck model with continuous distributions of values of time and schedule delay, but the ratio of the value of schedule delay time early to the value of schedule delay time late is homogenous. They concluded that congestion pricing can leave a majority of users better off even without returning the toll revenues to them. The consumer surplus losses or gains from tolling are not strictly monotonic in the value of time, because they are also dependent of the values of schedule delay. The greatest losses are not incurred by commuters with the lowest value of time. Similarly, Hall (2013) considered user preferences to be continuously distributed, but homogenous ratio of the costs of being late and being early. If a general joint distribution of these user-specific parameters is taken into account, obtaining an analytical equilibrium solution seems intractable. Xiao et al. (2013) studied the efficiency and effectiveness of a tradable credit system for managing the morning commute congestion, in which late-arrival is not allowed and the ratio of values of time and schedule delay is homogenous.

In this paper, we consider the user heterogeneity in value of time (VOT) and value of schedule delay early (VSDE). As done in Xiao et al. (2013), VOT and VSDE vary across all users. But, we let the ratio of VSDE to VOT also vary across all users. This consideration could stem from income differences of commuters. For example, rich people might be more flexible for arriving and thus have low VSDEs relative to their VOTs, whereas poor people may have tighter time budgets and then high VSDEs relative to VOTs (Koster and Koster, 2013). Furthermore, variations in VOT and VSDE over time (Tseng and Verhoef, 2008) are neglected. In other words, it is assumed that for arbitrary user, his/her VOT and VSDE are constant over time.

In the light of the difficulty of obtaining no-toll equilibrium solutions (Hall, 2013), we assume that arrival late is not permitted. In recent years, this assumption is adopted in many similar studies (e.g. Arnott and de Palma, 2011; de Palma and Arnott, 2012; Xiao et al., 2013; Xiao and Zhang, 2013). With these settings, we propose an ordinary differential equation formulation of the bottleneck model and derive the no-toll equilibrium departure pattern. Our approach is robust in the sense that it can be applied to all forms of VOT and VSDE, i.e., they can be continuous or discrete, deterministic or stochastic. Furthermore, we address the first-best tolling and subsequently examine its distributional impacts in the context of the heterogeneity. We first find the explicit sufficient condition for achieving a Pareto-improving without imposing specific functional forms on heterogeneity when the toll revenue is not allowed for returning to users. It is Pareto-improving when the gap between VOT and VSDE increases as the ratio of VSDE to VOT increases. Finally, the proposed approach is applied to deal with such user heterogeneity that both VOT and VSDE follow uniform distributions. The Pareto-improving condition in these settings is not only sufficient but also necessary. What's more, the individual gain from tolling strictly monotonically increases with VOT and VSDE. Outside this condition, the first-best toll scheme makes users with relatively higher VOTs and VSDEs better off and those with relatively lower values worse off. And, users with higher VOTs and VSDEs could gain more or lose less from tolling.

The remainder of our paper is organized as follows. In the next section, we formulate the no-toll equilibrium of a bottleneck-constrained highway with the heterogeneity and give the analytical departure patterns using the proposed ordinary differential equation formulation in some special cases of user heterogeneity. In Section 3, we investigate the first-best tolling and examine its welfare effects. Section 4 is dedicated to application of the proposed approach into a case in which