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Estimating exponential scheduling preferences

Katrine Hjorth [a,*], Maria Börjesson [b], Leonid Engelson [b], Mogens Fosgerau [a,b]

[a] Technical University of Denmark, Denmark
[b] Royal Institute of Technology, Sweden

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ABSTRACT

Different assumptions about travelers' scheduling preferences yield different measures of the cost of travel time variability. Only few forms of scheduling preferences provide non-trivial measures which are additive over links in transport networks where link travel times are arbitrarily distributed independent random variables: Assuming smooth preferences, this holds only for specifications with a constant marginal utility of time at the origin and an exponential or affine marginal utility of time at the destination. We apply a generalized version of this model to stated preference data of car drivers' route and mode choice under uncertain travel times. Our analysis exposes some important methodological issues related to complex non-linear scheduling models: One issue is identifying the point in time where the marginal utility of being at the destination becomes larger than the marginal utility of being at the origin. Another issue is that models with the exponential marginal utility formulation suffer from empirical identification problems. Though our results are not decisive, they partly support the constant-affine specification, in which the value of travel time variability is proportional to the variance of travel time.

1. Introduction

Measurement of travelers' trip scheduling preferences is important because they drive departure time choices (Vickrey, 1973) and underlie travelers' values of travel time and travel time variability (Noland and Small, 1995; Bates et al., 2001; Fosgerau and Karlström, 2010). The purpose of this paper is to empirically estimate scheduling preference specifications with a certain convenient property: That the cost of travel time variability is additive over independent traffic links. Our results lend some support to the simplest scheduling preference specification having this property, defined by a constant marginal utility of time at the origin and an affine marginal utility of time at the destination. This specification implies that the value of travel time variability is proportional to the variance of travel time.

A large class of scheduling preferences can be formally represented as time-dependent rates of utility derived at different locations. Vickrey (1973) formulated the departure time choice problem in terms of minimizing the sum of two integrals with integrands representing the marginal utility of time (MUT) at origin (H) and at destination (W) relative to the time spent traveling with both MUT varying by time of day. This formulation was further developed by Tseng and Verhoef (2008) and generalized to a two-trip chain by Jenelius et al. (2011).

The popular 'constant H – step W' scheduling formulation (hereafter referred to as const-step) proposed by Vickrey (1969) and Small (1982) is a special case of the model suggested by Vickrey (1973), where H is constant and W is a step function

* Corresponding author.
E-mail address: khj@transport.dtu.dk (K. Hjorth).

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with two fixed utility rates for time spent at the destination before and after the Preferred Arrival Time (PAT).[1] Noland and Small (1995) used expected utility theory to derive a reduced-form utility function based on const-step scheduling preferences, under the assumption that the travel time is exponentially or uniformly distributed and that travelers choose their departure time optimally. This reduced-form model included the cost of travel time variability (TTV), expressed by the standard deviation of travel time. Fosgerau and Karlström (2010) generalized this result to a general travel time distribution: They showed that, in a reduced-form model based on const-step scheduling preferences, the standard deviation of travel time is an appropriate measure for valuation of TTV, with a value depending on the shape of the travel time distribution. Fosgerau and Engelson (2011) showed that a scheduling formulation with affine H and W implies that the measure of TTV is proportional to the variance of travel time, with a coefficient independent of the distribution shape. Engelson (2011) derived explicit measures of the valuation of TTV when both functions H and W are second-order polynomials or exponential functions. These are probably the most general functions for H and W with a closed form expression for the measure of TTV.

The above-mentioned results concern car travel, where departure time choice is continuous. However, the framework has also been generalized to the case of a scheduled transit service: Both Fosgerau and Engelson (2011) and Engelson and Fosgerau (2011) (the latter considering scheduling formulations with constant H and affine or exponential W) extended their results to the scheduled service case considering costs of travel time and headway in settings with uncertain travel times and fixed headways. The scheduling framework has also been applied in analyses considering settings with known travel times (no TTV) and known or random headways: For known headways, Fosgerau (2009) derived the value of headway for general scheduling preferences and the case of const-step preferences. For random headways, Benezech and Coulombel (2013) derived values of (mean) headway and service reliability (standard deviation of headway) in the case of const-step preferences.

Travel time reliability is becoming increasingly important in transport modeling and assignment (Lam et al., 2014). In transport network applications, TTV and other components of the generalized travel costs are usually measured on links. In order to convert generalized travel cost from link to trip level, network assignment algorithms generally assume that travel costs are additive across links within each path. Hence, it is desirable that the measure of travel time variability is additive. Using the standard deviation of travel time as a measure of TTV does not satisfy this, as the standard deviation is additive only in the unrealistic case of perfect positive correlation between the link travel times. The variance, however, is additive if the link travel times are independent. The assumption of independence is strong, but results from Fosgerau and Fukuda (2012) and Eliasson (2007) suggest that the error made when converting the TTV from link to trip level while assuming independence of the travel time distributions across links is small in many cases.

Engelson and Fosgerau (2011) showed that the only form of smooth scheduling preferences providing an (non-trivial) additive measure of TTV when link travel times are arbitrarily distributed independent random variables, is a constant H and an exponential or affine W.[2] These specifications thus give a convenient expression of travel cost for calculation of the value of TTV in transport networks assuming that the distribution of travel times for different links are independent.

Even if scheduling models implying additive measures of TTV are desirable for application purposes, the most appropriate scheduling specification and corresponding measure is an empirical matter. No previous study has estimated the 'constant H – exponential W' (const-exp) scheduling model and tested its empirical properties. Most departure time choice modeling studies have applied the const-step formulation by Small (1982) and Abkowitz (1981), see the overview by de Jong et al. (2003). Hendrickson and Plank (1984) estimated quadratic penalty functions for early and late arrivals that correspond to a W function defined by an affine expression before PAT and another affine expression after PAT. Ettema et al. (2004) estimated parameters of the time varying utility rates based on departure time choice of complete tours while Tseng and Verhoef (2008) applied non-parametric techniques to estimate such rates for trips from home to work. There is also a vast body of empirical studies estimating the disutility of travel time variability directly, using measures of travel time variability such as standard deviation, variance or mean-delay, using SP data (see, e.g., the review in Carrion and Levinson, 2012). Estimating both a scheduling and reduced-form models, Börjesson et al. (2012) find that scheduling preferences do not capture the traveler's entire disutility of delays.

Very few scheduling models have been estimated on real data. The main contribution of this paper is showing that it is possible to empirically estimate scheduling functions that have the desirable additivity property, the const-exp specification, and its limiting case, the const-affine specification. Based on stated preference (SP) survey data we are also able to estimate the more general, exponential-exponential specification of H and W (exp-exp), suggested by Engelson (2011), and the conventional const-step specification. We attempt to evaluate whether the specifications providing additive measures for the travel time variability can be preferred to these benchmark models. We find that the parameters of the exponential scheduling functions H or W are poorly identified, with very high standard errors if the models can be estimated at all. However, applying multiplicative error terms facilitates the identification in many cases. It may be possible to design a SP experiment that allows identification of exponential scheduling functions with higher precision, but the appropriate scheduling function is usually not known when the SP experiments are designed. A second contribution of this paper is to reveal these methodological problems.

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[1] But actually, Vickrey's (1969) and Small's (1982) models were not formulated in terms of utility rates, but in terms of corresponding cost functions.

[2] The 'constant H – exponential W' formulation implies that the cost of TTV is proportional to the cumulant generating function $\ln E\exp(\beta T)$ where T is the random travel time and E is the expectation. As mentioned above, the 'constant H – affine W' formulation implies that the cost of TTV is proportional to the variance of travel time.