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A kinematic wave theory of capacity drop

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ABSTRACT

Capacity drop at active bottlenecks is one of the most puzzling traffic phenomena, but a thorough understanding of its mechanism is critical for designing variable speed limit and ramp metering strategies. In this study, within the framework of the kinematic wave theory, we propose a simple model of capacity drop based on the observation that capacity drop occurs when an upstream queue forms at an active bottleneck. Different from existing models, the new model still uses continuous fundamental diagrams but employs an entropy condition defined by a discontinuous boundary flux function, which introduces a traffic state-dependent capacity constraint. For a lane-drop area, we demonstrate that the model is well-defined, and its Riemann problem can be uniquely solved. After deriving the flow-density relations upstream and downstream to a bottleneck location, we find that the model can replicate the following three characteristics of capacity drop: the maximum discharge flow-rate can be reached only when both upstream and downstream traffic conditions are uncongested, capacity drop occurs when the bottleneck is activated, and some steady traffic states cannot be observed at both locations. We show that the new model is bistable subject to perturbations in initial and boundary conditions. With empirical observations at a merging bottleneck we also verify the three characteristics of capacity drop predicted by the new model. Through this study, we establish that the new model is physically meaningful, conceptually simple, computationally efficient, and mathematically tractable. We finally discuss future extensions and potential applications of the new model.

1. Introduction

Since the 1990s, the so-called two-capacity or capacity-drop phenomenon of active bottlenecks, in which “maximum flow rates decrease when queues form”, has been observed and verified at many bottleneck locations (Banks, 1990, 1991b; Hall and Agyemang-Duah, 1991). For example, at a merge bottleneck, when the total demand of the upstream mainline freeway and the on-ramp exceeds the capacity of the downstream mainline freeway, a queue forms on the mainline freeway, and the discharge flow-rate drops below the capacity of the downstream mainline freeway. Such “capacity drop” has also been observed at tunnels, lane drops, curves, and upgrades, where the bottlenecks cannot provide sufficient space for upstream vehicles (Cassidy and Bertini, 1999; Chung et al., 2007). Capacity drop also occurs at bottlenecks caused by work zones (Krammes and Lopez, 1994; Dixon et al., 1996; Jiang, 1999) as well as accidents/incidents (Smith et al., 2003).

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A drop in the downstream bottleneck's discharge flow-rate can further reduce the discharge flow-rates of impacted upstream off-ramps and the total discharge flow-rate of the whole corridor and, therefore, prolong vehicles' travel times (Newell, 1993; Daganzo, 1999). That the capacity of a road network may drop substantially when it is most needed during the peak period has been a baffling feature of freeway traffic dynamics (Papageorgiou and Kotsialos, 2002). Hence to prevent or delay the occurrence of capacity drop has been an important motivation and theoretical foundation for developing ramp metering, variable speed limits, and other control strategies (Banks, 1991a; Papageorgiou et al., 1991, 1997; Cassidy and Rudjanakanoknad, 2005; Papageorgiou et al., 2005, 2007).

Since 1960s, it has been observed that the flow-density relation, i.e., the fundamental diagram, can be a discontinuous function or multi-valued with a reverse-lambda shape (Edie, 1961; Drake et al., 1967; Koshi et al., 1983; Payne, 1984; Hall et al., 1992). This is different from traditional fundamental diagrams derived from car-following models in steady states, in which the flow-rate is a continuous function of the density. In (Hall and Agyemang-Duah, 1991; Hall et al., 1992), it was shown that discontinuous fundamental diagrams generally arise inside the bottleneck area and suggested that the discontinuity is associated with the capacity drop phenomenon. In the literature, many models of capacity drop have been based on the assumption of discontinuous fundamental diagrams. For example, in (Lu et al., 2008, 2009), an attempt was made to describe capacity drop with discontinuous fundamental diagrams within the framework of the LWR model (Lighthill and Whitham, 1955; Richards, 1956).

However, a discontinuous fundamental diagram is challenged both theoretically and empirically. Theoretically, a discontinuous flow-density relation is non-differentiable at the discontinuous point (usually the critical density) and leads to infinite characteristic wave speeds (Li and Zhang, 2013). Clearly this contradicts the fact that information travels at a finite speed along a traffic stream. Empirically, even though many studies confirm the existence of discontinuous fundamental diagrams inside a bottleneck area, e.g., Fig. 4 of (Hall et al., 1992), Cassidy (1998) demonstrated that, in near-stationary states, bivariate fundamental diagrams are still continuous at a location upstream to a bottleneck with capacity drop, but densities in some ranges cannot be observed.

Kinematic wave theories, e.g., the LWR model and the Cell Transmission Model (CTM) (Daganzo, 1995), have been powerful tools to analyze and simulate the queue formation, propagation, and dissipation processes through shock and rarefaction waves connecting different steady states. They have been widely used in designing ramp metering and other control strategies (Gomes and Horowitz, 2006). To the best of our knowledge, however, there has been no systematic theory of capacity drop with continuous fundamental diagrams.

In this study we propose a new model of capacity drop to reconcile continuous fundamental diagrams with capacity drop. For an active lane-drop bottleneck, as shown in Fig. 1, we attempt to replicate the observation that "maximum flow rates decrease when queues form" with the continuous CTM formulation of the kinematic wave theory developed in (Jin et al., 2009, 2012b), in which the junction flux function in terms of upstream demands and downstream supplies is used as an entropy condition to pick out unique, physical solutions. In particular, from CTM we can see that an upstream queue forms when the upstream demand is larger than the downstream supply. Then we introduce a new flux function based on the observation that upstream congestion and capacity drop occur immediately after the upstream demand exceeds the downstream supply. Here the new flux function is a discontinuous function in upstream demand and downstream supply. This is different from traditional flux functions, which are generally continuous (Daganzo, 1995; Lebacque, 1996; Jin and Zhang, 2003b; Ni and Leonard, 2005; Lebacque and Khoshyaran, 2005; Jin, 2010; Tampère et al., 2011; Jin, 2012b). With the new model we aim to reproduce the following characteristics of capacity drop: (i) when both upstream and downstream locations are uncongested, the discharge flow-rate can reach the downstream capacity; (ii) capacity drop occurs when the bottleneck is activated; i.e., when the upstream location is congested but downstream not, the discharge flow-rate drops below the downstream capacity; and (iii) some steady traffic states cannot be observed at both upstream and downstream locations, and the observed flow-density relations are discontinuous.

In the literature, there have been many studies on capacity drop. This study has a number of distinctive features.

- In contrast to existing kinematic wave models of capacity drop (Lu et al., 2008, 2009), the new model still uses continuous fundamental diagrams for the upstream and downstream links and is therefore devoid of unrealistic infinite information propagation speeds.[1] However, discontinuous flow-density relation can arise inside the bottleneck area where we can observe mixed congested and uncongested traffic states.
- The new model is still of the first order as the LWR model. In (Carlson et al., 2010; Parzani and Buisson, 2012), higher-order continuum models were shown to replicate capacity drop, but the capacity in higher-order models may be different from the generally used value in steady states (Zhang, 2001).
- The new model is phenomenological, different from (Leclercq et al., 2011), where the capacity drop magnitude was endogenously calculated by considering merging vehicles as moving bottlenecks. We assume that the magnitude of capacity drop is given or calibrated for a bottleneck. Such a model is much simpler and more suitable for system-level control and management applications.

[1] A characteristic wave, whose speed equals the derivative of flow-rate in density, can be considered as the information propagation wave of a small disturbance around a constant traffic density. Thus a discontinuous fundamental diagram leads to an infinite information propagation speed around the density where the flow-rate jumps.