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A day-to-day dynamical model for the evolution of path flows under disequilibrium of traffic networks with fixed demand

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ABSTRACT

Transportation networks are often subjected to perturbed conditions leading to traffic disequilibrium. Under such conditions, the traffic evolution is typically modeled as a dynamical system that captures the aggregated effect of paths-shifts by drivers over time. This paper proposes a day-to-day (DTD) dynamical model that bridges two important gaps in the literature. First, existing DTD models generally consider current path flows and costs, but do not factor the sensitivity of path costs to flow. The proposed DTD model simultaneously captures all three factors in modeling the flow shift by drivers. As a driver can potentially perceive the sensitivity of path costs with the congestion level based on past experience, incorporating this factor can enhance real-world consistency. In addition, it smoothens the time trajectory of path flows, a desirable property for practice where the iterative solution procedure is typically terminated at an arbitrary point due to computational time constraints. Second, the study provides a criterion to classify paths for an origin–destination pair into two subsets under traffic disequilibrium: expensive paths and attractive paths. This facilitates flow shifts from the set of expensive paths to the set of attractive paths, enabling a higher degree of freedom in modeling flow shift compared to that of shifting flows only to the shortest path, which is behaviorally restrictive. In addition, consistent with the real-world driver behavior, it also helps to preclude flow shifts among expensive paths. Improved behavioral consistency can lead to more meaningful path/link time-dependent flow profiles for developing effective dynamic traffic management strategies for practice. The proposed DTD model is formulated as the dynamical system by drawing insights from micro-economic theory. The stability of the model and existence of its stationary point are theoretically proven. Results from computational experiments validate its modeling properties and illustrate its benefits relative to existing DTD dynamical models.

1. Introduction

Transportation networks often face perturbed conditions due to diverse events such as traffic accidents, construction activities, work zones, and the opening of a new traffic link. The temporal extent of the network-related impacts of these perturbations can range from a few hours to several months. Under such conditions, some drivers find themselves on costlier paths compared to others. This disequilibrium of traffic motivates drivers to shift paths. Drivers on costlier paths shift to cheaper paths to reduce their individual travel cost, resulting in the evolution of traffic over time. Finally, a stationary state

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is reached in which there is no incentive to switch paths. This aspect raises an important modeling problem: how to represent the path-shift behavior of drivers under the disequilibrium of traffic networks? It has important implications for practice in terms of influencing the development of more effective strategies to mitigate/calm the disruptive effects of perturbation events. The problem is generally modeled as a dynamical system that captures the aggregated effect of path shifts of drivers in the form of a rate of change of network flows (path flows or link flows) over time.

The modeling of path-shift behavior of drivers depends on the underlying event that leads to the disequilibrium of traffic network. These events can be divided into four categories based on their impact on network flows and the time extent of the impact. The most common among them is the traffic incidents or accidents that affect the network flows for a short duration. In general, such transient phenomena can be captured by within-day dynamics (Ben-Akiva et al., 1991; Friesz et al., 1989, 1993). The second category is the demand variation due to special events such as a football game or a concert; in general, such events also have short-term effects and can be captured by modeling within-day dynamics. The third category of events is construction activities and work zones, they lead to capacity reductions and full/partial blockage of some links. Depending on the extent of such events, their impacts can last from a few days to several months. The evolution of traffic under these events can be captured through the modeling of the day-to-day (DTD) dynamics. The fourth category of events is topological changes in the network such as a bridge collapse (Zhu et al., 2010; He and Liu, 2012), opening of a bypass to traffic, and the restriction of right-of-way from two-way to one-way. The cascading effects of such phenomena on traffic patterns may also last for a long time and can be captured using DTD dynamics. This research focuses on modeling the DTD dynamics under fixed travel demand.

DTD dynamical models can be classified into two categories based on their day-to-day adjustment process: deterministic process models and stochastic process models. Deterministic process models lead to a single (link/path) flow vector at each time step. Stochastic process models (Cascetta, 1989; Davis and Nihan, 1993; Cantarella and Cascetta, 1995; Watling and Hazelton, 2003) determine a probability distribution of possible flow vectors at each time step. Hence, while the equilibrium of a deterministic process model refers to a single flow vector, the equilibrium of a stochastic process model implies a probability distribution of possible flow vectors. Deterministic process models are characterized by multiple domains of attraction, and model output is dependent on the starting point. By contrast, stochastic process models have a single domain of attraction, and hence the final state is independent of the starting point. An additional advantage of a stochastic process model is its ability to capture the variability associated with the random parameters of the DTD evolution (Watling and Cantarella, 2013). However, a challenge for its application is the need to determine an initial probability distribution. Recently, Smith et al. (2014) propose models that exhibit characteristics of both deterministic and stochastic process models.

The deterministic process DTD models can be classified further into two categories (Watling, 1999): deterministic route choice DTD models and probabilistic route choice DTD models. Deterministic route choice DTD models (Smith, 1984; He et al., 2010) assume that perceived and measured travel times of paths are equal and lead to a Wardropian equilibrium which is typically non-unique in terms of path flows but unique in terms of link flows. In addition, a deterministic path choice based on bounded rationality can lead to boundedly rational user equilibrium (Han et al., 2015). Probabilistic route choice DTD models (Horowitz, 1984; Cantarella, 1993; Cantarella and Cascetta, 1995) assume that perceived travel times of paths are different from their measured travel times and vary over individuals, leading to a stochastic user equilibrium. An attractive feature of probabilistic route choice DTD models is the property of unique equilibrium point. However, a key implementation challenge is the identification of the distribution function and related parameters for the error term in the perceived path travel costs. By comparison, deterministic route choice models are easier to implement but can have multiple equilibrium points. The proposed study seeks to develop a deterministic process DTD model based on deterministic route choice that leads to a Wardropian equilibrium which is a widely used objective for long-term planning in practice.

In past studies, the DTD evolution of transportation network (link/path) flows has been modeled using both analytical and simulation-based approaches. Yang and Zhang (2009) classify the analytical approaches into five categories: the simplex gravity flow dynamics (Smith, 1983), the proportional-switch adjustment process (Smith, 1984; Smith and Winsten, 1995; Huang and Lam, 2002; Peeta and Yang, 2003), the tatonnement process (Friesz et al., 1994), evolutionary dynamics (Sandholm, 2001; Yang, 2005) and the projected dynamical system (Zhang and Nagurney, 1995, 1996; Nagurney and Zhang, 1996, 1997). Several studies adopt simulation-based or field survey approaches to model the DTD dynamics (Mannering et al., 1990; Hu and Mahmassani, 1995; Mahmassani and Stephan, 1988; Caplice and Mahmassani, 1992; Mahmassani and Jou, 2000; Srinivasan and Mahmassani, 2000; Srinivasan and Guo, 2004). While these approaches consider several factors including the socioeconomic and weather-related variables to model the path-shift behavior, the analytical models typically rely on the path features for this purpose. There are three important path features that can influence the number of drivers shifting their path, namely, the current path cost, the current path flow and the sensitivity of the path cost to flow. To the best of the authors' knowledge, there is no DTD model that simultaneously considers these three path characteristics to model the path shift behavior under traffic disequilibrium. In general, most studies consider only the path costs and current path flows to determine the flow shift proportions. In a real-world scenario, if several cheaper paths are available, a few drivers would likely shift to a path whose cost increases rapidly with flow compared to the other paths. Ignoring this factor in the modeling process implies that drivers do not consider the sensitivity of path cost to the flow, thereby raising issues of behavioral consistency. In addition, for flow update at each time step, the existing DTD models typically shift flow from all used paths to the single shortest path, or from the set of paths with costs above average to the set of paths with costs below average, or based on pair-wise comparisons of costs of used paths connecting an origin to a destination. But a more customized approach would be to identify a set of attractive paths for each origin–destination (O–D) pair based on the