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The corridor problem with discrete multiple bottlenecks ☆

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ABSTRACT

This paper presents a transparent approach to the analysis of dynamic user equilibrium and clarifies the properties of a departure-time choice equilibrium of a corridor problem where discrete multiple bottlenecks exist along a freeway. The basis of our approach is the transformation of the formulation of equilibrium conditions in a conventional "Eulerian coordinate system" into one in a "*Lagrangian-like coordinate system*." This enables us to evaluate dynamic travel times easily, and to achieve a deep understanding of the mathematical structure of the problem, in particular, about the properties of the demand and supply (queuing) sub-models, relations with dynamic system optimal assignment, and differences between the morning and evening rush problems. Building on these foundations, we establish rigorous results on the existence and uniqueness of equilibria.

1. Introduction

Traffic congestion during morning and evening peak periods causes serious economic losses in most large cities. The Vickrey (1969) bottleneck model was proposed to understand and characterize this problem. It describes commuters' departure-time choice equilibrium at a single bottleneck and has been extensively studied by many researchers (e.g., Hendrickson and Kocur, 1981; Smith, 1984; Daganzo, 1985). The model has been applied to many problems because of its simplicity and clarity. For example, it has been used to evaluate traffic policies (e.g., Arnott et al., 1993a; Daganzo and Garcia, 2000; Akamatsu et al., 2006).

If there is more than one bottleneck, there are two streams of research for analyzing user equilibria. The first uses a straightforward generalization of Vickrey's model, whereas the second considers the dynamic user equilibrium (DUE) on a general network.

In the former stream of research, Kuwahara (1990) and Arnott et al. (1993b) analyzed an user equilibrium in two-tandem bottleneck networks, and Lago and Daganzo (2007) studied a similar problem where there are spillover and merging effects. These generalizations can provide valuable insights into the distribution of congestion among commuters with different origins. However, it is almost impossible to obtain an equilibrium in more general cases where a larger number of bottlenecks exist in a freeway corridor because this stream of studies has only used an analytical approach. Indeed, Arnott and DePalma

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(2011) studied a variant of the general problem, the so-called *corridor problem*, but they could not provide a complete equilibrium solution.

The latter stream of research applies a more computational approach. Although various mathematical formulations of DUE have been proposed (see, Szeto and Wong, 2011), they are considerably complex ways to analyze properties of an equilibrium because they must handle the complicated nested structure between link and path travel times in a network. Consequently, many issues remain regarding DUE properties, such as their existence, uniqueness, and stability (Iryo, 2013). Moreover, there is no algorithm that ensures convergence to a DUE solution, although many algorithms have been proposed. This is because most algorithms require *monotonicity* of the problem to guarantee convergence. However, this property does not hold for DUE problems, as we shall see later.

This paper presents a transparent approach for analyzing DUE and clarifies the properties of a departure-time choice equilibrium for a corridor problem where discrete multiple bottlenecks exist along a freeway. The basis of our approach is the transformation of the equilibrium conditions from a conventional "Eulerian coordinate system" into a "*Lagrangian-like coordinate system*." This is valuable for the morning rush (many-to-one travel demand) problem and the reverse problem in the evening (one-to-many travel demand problem). We can use this approach to easily evaluate the travel times without the abovementioned complication. We can then achieve deep insights into the mathematical structure of the problem, in terns of the properties of the demand and supply (queuing) sub-models, the relationships with dynamic system optimal assignment, and the differences between the morning and evening rush problems. Building on these foundations, we establish rigorous results on the existence and uniqueness of equilibria.

The remainder of this paper is organized as follows. In Section 2, we describe the dynamic user equilibrium for the corridor problem in an Eulerian coordinate system and discusses why it is difficult to analyze the problem in this form. Section 3 contains our reformulation of the equilibrium conditions in a Lagrangian-like coordinate system. To develop an understanding of the mathematical properties of the problem, we present a preliminary analysis for the deterministic demand case in Section 4. In Section 5, we examine the existence and uniqueness of equilibria. Section 6 provides numerical examples of the equilibrium flow patterns. Finally, Section 7 concludes the paper.

2. The corridor problem

2.1. Assumptions

Consider a freeway corridor consisting of N on-ramps (origin nodes) and a single off-ramp (destination node). These nodes are numbered sequentially from destination node 0 to the most distant on-ramp node N, as shown in Fig. 1. We assume that there is a single bottleneck i with a finite capacity μ_i in a segment between each pair $(i-1, i)$ of adjacent ramps. Therefore, there are N bottlenecks in the network numbered sequentially $(1, 2, \ldots, N)$ from downstream to upstream. At each bottleneck, a queue can form when the inflow rate exceeds the capacity. The queue evolution and the associated queuing delay are assumed to be represented by a point queue model (described in 2.2). The distance between each pair $(i-1, i)$ of adjacent bottlenecks measured by the free flow travel time is given by c_i.

From each on-ramp i $(i = 1, \ldots, N)$, Q_i commuters in residential location i $(i = 1, \ldots, N)$ enter the network and reach their destination during the morning rush-hour $\mathcal{T} \in [0, T]$. We assume that the travel demands $\{Q_i\}$ are given constants. Each commuter chooses the departure time of her/his trip from home (on-ramp) to workplace (destination) so as to minimize her/his disutility. This disutility includes travel time, queuing delay at bottlenecks, and a "schedule delay cost" that is associated with deviation from the wished arrival time to the destination. We assume that all commuters are homogeneous, such that they have the same desirable arrival time (work start time) t_w at the workplace, the same value of time, and the same penalty function for the schedule delay. However, for the arrival/departure-time choice principle, we employ a random utility model that describes the heterogeneity of users. The main problem considered in this paper is, under the assumptions mentioned above, to characterize a dynamic user equilibrium distribution of arrivals at the destination where no commuter could reduce her/his disutility by changing arrival times.

2.2. Arrival/departure time choice equilibrium

Under the assumptions above, we formulate a model that describes the equilibrium in the corridor network. At equilibrium, the following three conditions should hold.

2.2.1. Queuing conditions at each bottleneck

We describe the queuing congestion at each bottleneck in the network using a point queue model, where a queue is assumed to form vertically at the entrance of each bottleneck. The model can be represented by the following three conditions. First, the state equation for the number of users queuing at each bottleneck is

$$E_i(t) = A_i(t) - D_i(t), \quad \forall t \in \mathcal{T}, \ \forall i \in \mathcal{N}, \tag{2.1a}$$

where $A_i(t)$ $[D_i(t)]$ denotes the cumulative number of users who arrive at [depart from] bottleneck i by time $t, E_i(t)$ is the number of users queuing (existing) in bottleneck i at time t, and $\mathcal{N} \equiv \{1, \ldots, N\}$. Because a queue has no physical length