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Stochastic approximations for the macroscopic fundamental diagram of urban networks

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ABSTRACT

This paper proposes a theory for estimating the Macroscopic Fundamental Diagram (MFD) on inhomogeneous corridors and networks using probabilistic methods. By exploiting a symmetry property of the stochastic MFD, whereby it exhibits identical probability distributions in free-flow and congestion, it is found that the network MFD depends mainly on two dimensionless parameters: the mean block length to green ratio and the mean red to green ratio. The theory is validated with an exact traffic simulation and with the empirical data from the city of Yokohama. It is also shown that the effect of buses can be approximated with the proposed theory by accounting for their effect in the red to green ratio parameter.

1. Introduction

It has been shown experimentally by Geroliminis and Daganzo (2008) that the average flow on an urban network can be accurately predicted knowing the average density in the network. This urban-scale Macroscopic Fundamental Diagram (MFD) appears as an invaluable tool to overcome the difficulties of traditional planning models. Although it is still under debate whether it depends on trip origins and destinations and route choice, there is no question that network topology and control parameters such as block length, existence of turn-only lanes, and traffic light settings play a key role (Ramezani et al., 2015; Yildirimoglu and Geroliminis, 2014; Haddad and Shraiber, 2014; Zhang et al., 2013).

Existing methods to estimate the MFD analytically for simple homogeneous arterial corridors can be categorized into three types: (i) empirical (Geroliminis and Daganzo, 2007, 2008; Wu et al., 2011; Saberi and Mahmassani, 2012; Geroliminis and Sun, 2011; Geroliminis and Ji, 2011; Cassidy et al., 2011; Knoop et al., 2015; Gayah and Daganzo, 2011; Buisson and Ladier, 2009; Daganzo et al., 2011), (ii) analytical (Daganzo and Geroliminis, 2008; Leclercq et al., 2014), and (iii) simulation (Ji et al., 2010; Mazlomian et al., 2010; Geroliminis and Boyacı, 2013; Haddad and Geroliminis, 2012; Haddad et al., 2013; Knoop and Hoogendoorn, 2013; Knoop et al., 2013; Gayah et al., 2014). Existing analytical results are based on the method of cuts for homogeneous corridors, i.e. with equal block size, signal settings and constant offset, and therefore, one can focus on the cuts from a single intersection to compute the MFD for the whole corridor. Despite this apparent simplicity, this approach quickly becomes intractable, more so if buses are introduced (Chiabaut et al., 2014; Chiabaut, in press). Even with no buses the homogeneous corridor method cannot be scaled up without complications to estimate the network MFD mainly because a network path cannot be guaranteed to have constant offset all along, even for homogeneous networks.

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To overcome these difficulties, in this paper we introduce the concept of stochastic corridors, where any particular inhomogeneous corridor – with different block lengths and signal timing – is seen as a particular realization. Stochastic corridors are in fact probabilistically homogeneous in the sense that the distribution of these network parameters does not change in time or space. This approach allows the estimation of the network MFD, for which analytical methods are currently unavailable.

This paper is organized as follows. Section 2 develops the theory of stochastic corridors, which is based on renewal theory. The existence of short blocks is examined in detail in Section 3, as it can severely reduce corridor capacity. Section 4 is devoted to comparing the theory both with an exact traffic simulation and the empirical data from the city of Yokohama presented in Geroliminis and Daganzo (2008). Finally, Section 5 presents a discussion.

2. Stochastic corridors

Consider an inhomogeneous corridor consisting of a large sequence of road segments of different length, each one delimited by a traffic signal with settings that vary in time and across segments. This particular corridor is viewed here as a realization of a "stochastic corridor" random variable, where the length of each segment and the red and green times of its signals are random variables ℓ, r and g, respectively, assumed to be independent. We use the symbols μ, σ and $\delta = \sigma/\mu$ for the mean, standard deviation, and coefficient of variation of a random variable, whose name will be indicated as subscript; e.g., block lengths are assumed i.i.d. with mean and variance μ_ℓ, σ_ℓ^2. Turning movements are not considered in our analysis.

We use the superscripts $\sharp$ and $\flat$ to differentiate variables pertaining to forward and backward cuts, respectively, while the superscript $-$ will be used as their placeholder. All links in the network are assumed to obey a triangular fundamental diagram (FD) with free-flow speed $w^\sharp$, wave speed $-w^\flat$ and jam density κ; the saturation flow is therefore $Q = \kappa w^\flat w^\sharp/(w^\flat + w^\sharp)$.

Our formulation is based on variational theory Daganzo (2005a,b), which corresponds to the solution of the kinematic wave model of Lighthill and Whitham (1955);Richards (1956) when expressed as a Hamilton–Jacobi partial differential equation. This solution – known as the Hopf-Lax formula (Lax, 1957; Hopf, 1970) – states that the number of vehicles that have crossed location x by time $t, N(t,x)$, can be expressed in variational form as:

$$N_P = \inf_{B \in \mathcal{B}_P} \{N_B + \Delta_{BP}\} \tag{1}$$

where P is a generic point with coordinates $(t,x), \mathcal{B}_P$is the set of all points in the boundary that are in the domain of dependence of P, the point $B \equiv (t_B, x_B)$ is in $\mathcal{B}_P, N_P \equiv N(t,x)$ and $N_B \equiv N(t_B, x_B)$, and Δ_{BP} is the "cost" or maximum number of vehicles that can cross the minimum path joining B and P; see Fig. 1a. (Notice that in the absence of bottlenecks, such as traffic lights, all valid paths – including the minimum path – between B and P have the same cost and it is customary to compute Δ_{BP} along the straight line BP.)

To derive the corridor MFD, consider the initial value problem in Fig. 1b where the vehicle number $N(t_B, x_B)$ is known in the boundary $t_B = 0$ such that the density, k, is constant. Noting that in this case, $N_B = N_O + (x - x_B)k$ with $N_O \equiv N(0,x)$, we can write

$$N_P - N_O = \min_B \{\Delta_{BP} + (x - x_B)k\}. \tag{2}$$

The MFD is defined as the steady-state flow at any location x; i.e.:

$$q(k) \equiv \lim_{t\to\infty} \frac{1}{t}(N_P - N_O) \tag{3a}$$

$$= \min_B \{\lim_{t\to\infty} \frac{1}{t}(\Delta_{BP} + (x - x_B)k)\}, \tag{3b}$$

$$= \min_v \{\phi(v) + vk\}. \tag{3c}$$

Expression (3c) is the method of cuts (Daganzo and Geroliminis, 2008), where $v \equiv (x - x_B)/t$ is the average speed of the cut and $\phi(v) \equiv \Delta_{BP}/t$ is its maximum passing rate. This method has proven very useful for deterministic homogeneous problems where $\phi(v)$ can be evaluated analytically.

In non-homogeneous corridors the main difficulty is identifying all valid paths between B and P and selecting the one with minimum cost. This problem is exacerbated when the corridor contains short blocks because the minimum path becomes increasingly difficult to calculate. To tackle this problem, this paper proposes using renewal theory to approximate (3b).

When the corridor does not contain short blocks we propose the following approximation. Much as in the method of cuts, the multitude of valid paths in a network is replaced by a small number of "observers" that wander through the network with simple rules with the hope that they will capture the most important valid paths. In our case, we add the restriction that observers must be statistically independent because otherwise the problem becomes intractable.

In the proposed method, observers are emanated at time t and travel backwards in time in a series of "renewal cycles" generated by a strategy, s, until reaching the boundary at some location $x_B = x_B^s$, while having crossed a maximum number of vehicles of $\Delta_{BP} = \Delta_{BP}^s$. In each renewal cycle, the observer starts at the beginning of a red phase, travels for a number of