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The split-demand one-commodity pickup-and-delivery travelling salesman problem

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ABSTRACT

This paper introduces a new vehicle routing problem transferring one commodity between customers with a capacitated vehicle that can visit a customer more than once, although a maximum number of visits must be respected. It generalizes the capacitated vehicle routing problem with split demands and some other variants recently addressed in the literature. We model the problem with a single commodity flow formulation and design a branch-and-cut approach to solve it. We make use of Benders Decomposition to project out the flow variables from the formulation. Inequalities to strengthen the linear programming relaxation are also presented and separated within the approach. Extensive computational results illustrate the performance of the approach on benchmark instances from the literature.

1. Introduction

In the capacitated Vehicle Routing Problem (VRP) there are products originally in a depot that must be delivered to a set of customers by an homogeneous fleet of vehicles. Since all products depart from the same source location, they can all be considered as *one commodity*. A route consists of paths, starting from and ending at the depot. Each path is called here *trip*. The trips in a route can be performed in parallel by a set of identical vehicles (each vehicle performs a different trip) or in sequence by a single vehicle. Each customer is a delivery location that must be visited once, while the depot is a pickup location. The load of the vehicle through a trip should never exceed its capacity. The aim of the VRP is to compute a route to satisfy the demand of each customer while minimizing the travel cost. See e.g. Toth and Vigo (2014) for a textbook on VRP and variants. In the Split Delivery Vehicle Routing Problem (SDVRP) the demand of a customer is allowed to be served in more than one visit, thus allowing routes with smaller travel cost. See e.g. Archetti and Speranza (2008) for a survey on SDVRP and related problems. In pickup-and-delivery problems the customers are also allowed to be pickup locations, and therefore it is important to distinguish between one-commodity and multi-commodity variants. See e.g. Berbeglia et al. (2007) and Parragh et al. (2008) for surveys on static problems in this area. Our paper merges the three topics in the following problem.

A finite set of locations is given and the travel cost from one location to another location is assumed to be known. One specific location is considered to be a depot and the other locations are identified as customers. Each customer requires or provides a given demand of a single commodity (the product). A product unit collected from a pickup location can be supplied to any delivery location. It is assumed that there is a vehicle with a given capacity, originally at the depot, to serve

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the demands of all customers through a route. A route is a set of trips that cover each customer *at least* once. While following the route, the vehicle can either deliver or collect product in each location. All the visits to a customer must end up with exactly its required demand. The number of times that the vehicle visits a customer is limited by a given parameter m. The *Split-Demand One-Commodity Pickup-and-Delivery Travelling Salesman Problem* (SD1PDTSP) is the problem of finding a min-cost route for the capacitated vehicle such that it satisfies the demand of all customers.

In this paper, the initial load of the vehicle when leaving the depot is a decision that must be computed within the optimization problem. Also in this paper, the depot is treated as a customer, providing (or absorbing) the sum of the customers' demands so the balance of the commodity in the system is zero. As it will be discussed later, the model and algorithm proposed in this paper for the SD1PDTSP can easily be adapted to several variants, including the one where the vehicle is required to leave the depot with full (or empty) load, the one where customers with zero demand may be not visited, and the one where not all customers must be visited and the demand of the depot is a decision variable.

As any customer, the depot in our problem definition is allowed to be visited several times by the vehicle through the route. The number of visits to the depot is the number of trips in the route, and is limited to at most a given parameter k. For simplicity in notation, we assume $k = m$, although in some applications the value k could be desired to be different than m.

Although it could make sense to require that product collected in a trip must be delivered in the same trip, we do not impose this requirement in the SD1PDTSP. Therefore all trips in a SD1PDTSP route are considered to be performed by a single vehicle sequentially. The problem name contains the words "Travelling Salesman Problem" (TSP) to emphasize this assumption. When the depot is the only pickup location, as on VRP instances, or when the vehicle is forced to leave the depot with full load, then the trips may be executed in parallel when a fleet of k identical vehicles are available. In general, however, product collected from a customer may go through the depot before being delivered to another customer, and therefore the trips cannot be performed in parallel by different vehicles. We discuss later how to adapt the mathematical model to ensure that product collected in a trip are also delivered in the same trip.

In the SD1PDTSP each location is assumed to have a known inventory of the product before starting the vehicle service. Also, each location is associated with a desired inventory that it must have after the last vehicle service. The difference between the inventories is the demand of the location. Inventories in a location can be reduced or augmented during intermediate visits of the vehicle with the only constraint of considering a given capacity associated with the location. In other words, we allow preemption in the SD1PDTSP, i.e. product units collected in a location can be unloaded (fully or partially) at any intermediate location to be picked up later and delivered in another location. We will observe how to adapt the model and algorithm described in this paper for the non-preemptive variant. We do not consider inventory holding cost in the SD1PDTSP. See e.g. Coelho et al. (2014) for a recent survey on the Inventory–Routing Problem.

Fig. 1 shows optimal routes for three problems based on a benchmark instance introduced in Mosheiov (1994). Each circle represents a customer and is located using given coordinates. The travel cost between two customers is defined as the Euclidean distance. The demand of a customer is the value near the circle representing that customer. Positive values are associated with pickup customers and negative values with delivery customers. The vehicle capacity is assumed to be 7. The load of the vehicle when leaving customer 1 (depot) is 6. Fig. 1(a) and (c) correspond to the SD1PDTSP with $m = 1$ and $m = 2$, respectively. Fig. 1(b) corresponds to the SD1PDTSP with $m = 2$ and with the additional constraint that preemption of the product in a customer is not allowed. These figures show that the three problems on the same data may have different optimal objective values, thus the travel cost may be reduced by allowing split-demand and preemption.

A practical application of SD1PDTSP arises in the context of a self-service bike-sharing system, where every night a capacitated vehicle visits all the bike stations in the district of a city to move bikes and restore the initial configuration of the system (see Chemla et al., 2013; Raviv et al., 2013; Dell'Amico et al., 2014).

This paper is organized as follows. Section 2 shows the relation between SD1PDTSP and other problems addressed in the literature. Section 3 presents a mixed integer linear programming formulation for SD1PDTSP and discusses several minor modifications to also model other problem variants. Section 4 describes a branch-and-cut algorithm based on projecting the continuous variables and on some new valid inequalities. Section 5 analyzes computational results obtained by applying the algorithm to solve SD1PDTSP, SDVRP and VRP instances.

2. Related problems

When the number of visits to a location is unbounded, the uncapacitated variant of the SD1PDTSP becomes the *Graphical TSP* (see e.g. Naddef and Rinaldi, 1992), where the aim is to find a min-cost route visiting each location at least once. When the number of visits to a location is bounded, the uncapacitated variant of the SD1PDTSP is a particular case of the *Generalized TSP* (see e.g. Fischetti et al., 1997), where each location is represented by a cluster of nodes (visits to a customer) in a graph. When the maximum number of visits to a location is one, the uncapacitated variant is the TSP.

The VRP can be considered as a SD1PDVRP where the customers are delivery locations, the depot is the only pickup location, and split is allowed only at the pickup location. In standard VRP instances, each customer demand is not larger than the vehicle capacity and the total delivery demand is larger than the vehicle capacity. Then, the depot must be visited several times (or equivalently, several trips are necessary). Some VRP articles assume that the number of trips must be the optimal objective value k of a Bin Packing Problem defined by the customer demands and the vehicle capacity. In other VRP articles, k