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Formulation, existence, and computation of boundedly rational dynamic user equilibrium with fixed or endogenous user tolerance

Ke Han [a,*], W.Y. Szeto [b], Terry L. Friesz [c]

[a] Department of Civil and Environmental Engineering, Imperial College London, United Kingdom
[b] Department of Civil Engineering, The University of Hong Kong, China
[c] Department of Industrial and Manufacturing Engineering, Pennsylvania State University, USA

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ABSTRACT

This paper analyzes *dynamic user equilibrium* (DUE) that incorporates the notion of *boundedly rational* (BR) user behavior in the selection of departure times and routes. Intrinsically, the *boundedly rational* dynamic user equilibrium (BR-DUE) model we present assumes that travelers do not always seek the least costly route-and-departure-time choice. Rather, their perception of travel cost is affected by an indifference band describing travelers' tolerance of the difference between their experienced travel costs and the minimum travel cost. An extension of the BR-DUE problem is the so-called *variable tolerance dynamic user equilibrium* (VT-BR-DUE) wherein endogenously determined tolerances may depend not only on paths, but also on the established path departure rates.

This paper presents a unified approach for modeling both BR-DUE and VT-BR-DUE, which makes significant contributions to the model formulation, analysis of existence, solution characterization, and numerical computation of such problems. The VT-BR-DUE problem, together with the BR-DUE problem as a special case, is formulated as a variational inequality. We provide a very general existence result for VT-BR-DUE and BR-DUE that relies on assumptions weaker than those required for normal DUE models. Moreover, a characterization of the solution set is provided based on rigorous topological analysis. Finally, three computational algorithms with convergence results are proposed based on the VI and DVI formulations. Numerical studies are conducted to assess the proposed algorithms in terms of solution quality, convergence, and computational efficiency.

1. Introduction

This paper studies an extension of the *simultaneous route-and-departure-time dynamic user equilibrium* (SRDT DUE) (Friesz et al., 1993). We incorporate the concept of *bounded rationality* (BR) proposed by Simon (1957, 1990, 1991) for the modeling of travel behavior. As such, BR-DUE models are developed under the assumption that travelers, viewed as Nash agents, do not behave in a completely rational manner.

* Corresponding author.
E-mail addresses: k.han@imperial.ac.uk (K. Han), ceszeto@hku.hk (W.Y. Szeto), tfriesz@psu.edu (T.L. Friesz).

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In the literature of traffic user equilibrium, the modeling of travelers' route and/or departure time choices has been greatly influenced by Wardrop's first principle (Wardrop, 1952), which states that road users behave in a rational way and seek to minimize their own travel times/costs by making route (or departure time) choices. There are multiple means of expressing the dynamic notion of Wardropian user equilibrium, such as variational inequality (Friesz et al., 1993; Han et al., 2013c; Szeto and Lo, 2004), differential variational inequality (Friesz et al., 2001, 2011, 2013b), and nonlinear complementarity problem (Han et al., 2011; Wie et al., 2002). While enjoying a number of canonical mathematical representations, the notion of completely rational user equilibrium is not entirely in line with realistic driving behavior and empirical observations. That is, travelers may not always choose the departure time and/or route that yield the minimum travel cost. Such behavior may be due to (1) imperfect travel information; and (2) certain "inertia" in decision making. Indeed, empirical studies suggest that in reality, drivers do not always follow the least costly route-and-departure-time choice (Avineri and Prashker, 2004). Accordingly, one may model such boundedly rational behavior by introducing travelers' tolerances towards the difference in travel costs, and postulating a range of costs acceptable to travelers, rather than the minimum travel cost.

The main subjects of this paper include *boundedly rational dynamic user equilibrium* (BR-DUE), which relies on fixed (exogenous) tolerances that often depend on the origin-destination (O-D) pair in the literature. We also consider the so-called *variable tolerance boundedly rational dynamic user equilibrium* (VT-BR-DUE), in which the tolerances may depend on the path and the established (actual) path departure rates. Clearly, BR-DUE is simply a special case of VT-BR-DUE. The latter concept is relevant in situations where drivers' tolerances may be affected by not only their O–D pairs (or more generally, their static attributes such as socio-economic status) but also by a number of external factors. For example, these tolerances may vary by path, depending on road quality, travel distance, scenic quality (Ben Akiva et al., 1984), and personal familiarity (Bonsall, 1992), all of which are associated with a given link or path. In addition, the user tolerances may depend on prevailing traffic conditions (Ueberschaer, 1971; Huchingson et al., 1977), which are completely determined by the actual path departure rates. However, it is important for us to clarify that whether and how the tolerances should depend on all the aforementioned factors is not the focus here. Rather, this paper is meant to provide the most general modeling framework that can accommodate all these factors if and when they become relevant. According to the literature review presented below, the notion of bounded rationality has received much attention in (dynamic) traffic assignment; and our current paper makes a significant contribution to this area by providing the first complete theory of BR-DUE and VT-BR-DUE.

1.1. Some literature on bounded rationality

As a relaxation of the perfect rationality assumption commonly made in Nash games, the notion of bounded rationality was first proposed by Simon (1957, 1990, 1991) and introduced to traffic modeling by Mahmassani and Chang (1987). In prose, the notion of bounded rationality postulates a range of acceptable travel costs that, when achieved, do not incentivize travelers to change their departure times or route choices. Such a range is phrased by Mahmassani and Chang (1987) as "indifference band". The width of such a band, usually denoted by ε, is either derived through a behavioral study of road users (for example, by surveys) or calibrated from empirical observation through inverse modeling techniques. In general, ε could depend on a specific origin–destination pair and/or travel commodity. Since Mahmassani and Chang (1987), boundedly rational user equilibrium (BR-UE) has received considerable attention in *static traffic assignment* (STA), with an incomplete list of research papers including Di et al. (2013), Gifford and Checherita (2007), Han and Timmermans (2006), Khisty and Arslan (2005), Lou et al. (2010) and Marsden et al. (2012). It is also investigated using simulation-based approaches in the venue of dynamic modeling (Hu and Mahmassani, 1997; Mahmassani and Jayakrishnan, 1991; Mahmassani and Liu, 1999; Mahmassani et al., 2005).

The notion of bounded rationality (BR) was first studied in a so-called laboratory setting by Mahmassani and Chang (1987), without a mathematical articulation of BR for dynamic traffic assignment. BR was used in a similar fashion for simulations by Jayakrishnan and Mahmassani (1990), Peeta and Mahmassani (1995), Mahmassani and Liu (1999), Chiu and Mahmassani (2002). Recognizing the lack of a theory of dynamic traffic assignment that directly incorporates BR, Ridwan (2004) tried to apply the theory of fuzzy systems to the study of BR. Bogers et al. (2005), again driven by the lack of a suitable theory, conducted more laboratory studies of BR. Szeto (2003) and Szeto and Lo (2006) proposed a mathematical model for *route-choice* (RC) boundedly rational dynamic user equilibrium (RC BR-DUE). The RC BR-DUE is formulated as a discrete-time nonlinear complementarity problem in Szeto and Lo (2006), where a heuristic route-swapping algorithm was proposed to solve the problem. Ge and Zhou (2012) considered RC BR-DUE with endogenously determined tolerances by allowing the width of the indifference band ε to depend on time and the actual path departure rates. However, no solution existence or computational method are provided in that paper. Contributions by Szeto (2003), Szeto and Lo (2006) and Ge and Zhou (2012) achieve enhanced (yet partial) integration of BR and DUE but do not establish and analyze a complete theory, where by "complete" we mean a mathematical formulation consistent with known empirical results and surmised behaviors; qualitative properties; and a computational approach that is demonstrably effective.

To the best of our knowledge, there has not been an analytical treatment of the BR-DUE problem in the literature, in terms of formulation, qualitative analyses, and computation. This paper bridges this gap by establishing the first complete analytical framework capable of formulating BR-DUE problems into familiar mathematical forms (such as variational inequalities),