Transportation Research Part B 71 (2015) 182–193

Contents lists available at ScienceDirect

Transportation Research Part B

journal homepage: www.elsevier.com/locate/trb

The kinematic wave model with finite decelerations: A social force car-following model approximation

Rafael Delpiano [a,*,1], Jorge Laval [b], Juan Enrique Coeymans [a], Juan Carlos Herrera [a]

[a] Departamento de Ingeniería de Transporte y Logística, Escuela de Ingeniería Pontificia Universidad Católica de Chile, Vicuña Mackenna 4860, Macul, Santiago, Chile

[b] School of Civil and Environmental Engineering, Georgia Institute of Technology, Mason Building, 790 Atlantic Drive, Atlanta, GA 30332, USA

ARTICLE INFO

Article history:
Received 12 May 2014
Received in revised form 10 October 2014
Accepted 10 October 2014
Available online 27 November 2014

Keywords:
Traffic flow theory
Microscopic traffic models
Social force model
Car-following
Kinematic wave model

ABSTRACT

This paper derives a five-parameter social force car-following model that converges to the kinematic wave model with triangular fundamental diagram. Analytical solutions for vehicle trajectories are found for the lead-vehicle problem, which exhibit clockwise and counter-clockwise hysteresis depending on the model's parameters and the lead vehicle trajectory. When coupled with a stochastic vehicle dynamics module, the model is able to reproduce periods and amplitudes of stop-and-go waves, as reported in the field. The model's stability conditions are analysed and its trajectories are compared to real data.

1. Introduction

Realistic vehicle accelerations (and decelerations) are becoming an increasingly important requirement of traffic flow models, mainly due to their use in emissions and/or safety studies. Currently, when selecting a traffic flow model there appears to be a trade-off between realistic accelerations and analytical tractability.

On the one hand, there are very simple models (Newell, 1993, 2002) that give exact analytical solutions but infinite accelerations. Although four-parameter, bounded-acceleration versions have been proposed (Laval and Daganzo, 2006), decelerations remain unbounded.

On the other hand, there are more complex models, such as the Intelligent Driver Model (*IDM*, Treiber et al., 2000) or the one by Gipps (1981), that give realistic accelerations but can only be solved numerically.

Therefore, it seems appealing to have a simple model that can be solved analytically and that has bounded acceleration and deceleration. This paper tries to fill this gap through a social force model.

In "social" or "generalized forces" models, vehicles are modelled as passive bodies subject to forces representing real-world driver behaviour. Social forces for vehicles were proposed sixteen years ago (Helbing and Tilch, 1998), and there is current research on them (e.g. Schönauer et al., 2012; Fellendorf et al., 2012). For a complete review on social force-based models, we suggest the work of Li and Sun (2012).

Tampère (2004) proposed a linear ("Helly") model (Helly, 1959) with an upper limit to acceleration in the form of a free-flow term. This paper arrives to an equivalent formulation of a particular case of this model through a social-force approach, and studies some of its characteristics in depth.

* Corresponding author.
E-mail addresses: delpiano@ing.puc.cl (R. Delpiano), jorge.laval@ce.gatech.edu (J. Laval), jec@ing.puc.cl (J.E. Coeymans), jch@ing.puc.cl (J.C. Herrera).

[1] Conicyt scholarship holder.

http://dx.doi.org/10.1016/j.trb.2014.10.005
0191-2615/

Although the simplicity of the derived model may make it less accurate than some of the state-of-the-art models, it offers some analytical advantages.

The formation of stop-and-go waves is a well documented feature of real traffic (cf. Treiber and Kesting, 2011), and thus a desirable feature of traffic models. The ability of a model to form and propagate stop-and-go waves depends on its stability (i.e. whether modelled follower vehicles amplify or reduce perturbations to their leader's trajectories). Wilson (2008) provided a stability analysis framework for a large family of models, as a function of their parameters. On the other hand, adding a random component to vehicles' acceleration has proven to enable some models to reproduce realistic stop-and-go waves (Laval et al., 2014).

The remainder of this article is organized as follows:

On Section 2 we derive a social force model with finite acceleration and deceleration, taking five parameters, showing some of its main features, as well as the computation of its macroscopic characteristics. On Section 3 we provide some analytical solutions to the model and an analysis of its stability. Following that, on Section 4, we reproduce stop-and-go waves on a simulation experiment. Finally, Section 5 specifies the main conclusions of the present work.

2. Derivation of the model

In a social force-model, driver behaviour is determined by the sum of all forces exerted over the vehicle. Helbing and Tilch (1998) propose two forces: (i) The acceleration force, f^a and (ii) an interaction or repulsive force, f^r. The total force acting on a particular vehicle i, is:

$$\dot{v}_i(t) = f_i^a(t) + f_i^r(t).$$

The acceleration force reflects the tendency of drivers to accelerate to their desired speed. As in previous models (e.g. Helbing and Tilch, 1998) it is parsimoniously defined as proportional to the difference between the current speed, v_i and the maximum desired speed, V, i.e.:

$$f_i^a(t) = (V - v_i(t))c_1, \tag{1}$$

where c_1 is a parameter in units of time^{-1}.

Helbing and Tilch (1998) propose a formulation for the interaction or repulsive force that obscures the nature of acceleration force by subtracting the desired speed. In contrast, our proposed repulsive force depends exclusively on a vehicle and its leader's dynamics. It becomes *active* only when a vehicle i is approaching or already near to its leading vehicle, $i-1$. This force grows linearly with the approach speed and with vehicle spacing $y_{i-1} - y_i$:

$$f_i^r(t) = \{(v_{i-1}(t) - v_i(t))c_2 + [y_{i-1}(t) - y_i(t) - \bar{s}(v_i(t))]c_3\}^-, \tag{2}$$

where $\{\cdot\}^-$ is equivalent to $\min\{\cdot, 0\}$, i.e. the value becomes zero if the leader is too far apart or not approaching fast enough. We will say that the repulsive force is *active* when it has a negative value. Symbols c_2 and c_3 are parameters of the model, in units of time^{-1} and time^{-2} respectively.

The term:

$$\bar{s}(v_i(t)) = \tau_r v_i(t) + s_r$$

is the "repulsive spacing" (i.e. a theoretical equilibrium spacing if there were no acceleration force) as a function of speed, where τ_r and s_r are parameters of the model in units of time and distance respectively. As we will see, the actual spacing usually differs, even in equilibrium.

As a result, the acceleration of vehicle i equals the total force exerted over it:

$$\dot{v}_i = (V - v_i)c_1 + \{(v_{i-1} - v_i)c_2 + (y_{i-1} - y_i - \tau_r v_i - s_r)c_3\}^-. \tag{3}$$

Note that under free-flow the repulsive force will be inactive (zero), while active only under congestion.

Eq. (3) is equivalent to:

$$\dot{v}_i(t) = min\{(V - v_i(t))c_1, (\Delta v_i(t))c_2 + (\Delta y_i(t) - \tau_m v_i(t) - s_m)c_3\}, \tag{4}$$

where:

$$\tau_m = \tau_r + \frac{c_1}{c_3}, \tag{5}$$

$$s_m = s_r - V\frac{c_1}{c_3}. \tag{6}$$

Which maps exactly to the model by Tampère (2004) with no quadratic dependence on speed (as cited by Ossen and Hoogendoorn (2008)), $T_r = 0$ and no multi-anticipation. The congested term is equivalent to a "linear" model (Helly, 1959) with no influence of the acceleration in spacing.

There is yet a special case of the model were parameter c_2 may be removed by making it a function of the others, if we note that the repulsive force can be seen as a damped harmonic oscillator system. To avoid spurious instabilities we would