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# Mathematical programming formulations for transit network design

Héctor Cancela, Antonio Mauttone *, María E. Urquhart

*Operations Research Department, Universidad de la República, J. Herrera y Reissig 565 (Instituto de Computación), Montevideo, Uruguay*

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ABSTRACT

In this work, we study the transit network design problem from the perspective of mathematical programming. More precisely, we consider the problem of defining the number and itinerary of bus routes and their frequencies, for a public transportation system. In this problem, the routes should be defined in terms of a given infrastructure of streets and stops and should cover a given origin–destination demand. The solution (routes and frequencies) should be convenient for the users and the operators. We review existing mathematical programming formulations and propose a new one, paying attention to the following aspects of public transportation systems, that are identified as key elements in order to have a realistic model: (a) the interest of the users, (b) the interest of the operators, (c) the behavior of the users, and (d) constraints regarding transfer, infrastructure and bus capacity. First, we discuss the formulations existing on the literature, in terms of the aspects mentioned above. Second, we propose a mixed integer linear programming (MILP) formulation, that incorporates the waiting time and the existence of multiple lines in the behavior of the users. We validate the proposed formulation using several cases, including a real one. Also, we compare the obtained results against results from the existing literature. In order to include transfer, infrastructure and bus capacity constraints, we propose an extension to the formulation and we discuss its impact in the structure of the model, based on concepts of bi-level mathematical programming. The mathematical formulations developed contribute towards a more realistic modeling effort, taking into account important aspects of the real system which were not included in previous proposals in the literature.



## 1. Introduction

Public transportation systems are present in most cities of the world, either conceived as a service that should be provided to the inhabitants, as a tool for urban planning, or as business of private companies. Such a system is composed by an infrastructure (usually built by the government) and services (provided by operators) which are offered to persons (users) that need to travel along the city. The design of a sustainable and efficient system from the viewpoint of the whole society entails the minimization of fixed and variable costs, over a set of many alternatives defined in terms of different variables. The complete design of the system poses a very complex problem, if it is approached as a single monolithic unit. Therefore, the problem is divided in parts of smaller size, in such a way that the resulting sub-problems can be tractable. A commonly

* Corresponding author. Tel.: +598 2 711 4244.
*E-mail addresses:* cancela@fing.edu.uy (H. Cancela), mauttone@fing.edu.uy (A. Mauttone), urquhart@fing.edu.uy (M.E. Urquhart).

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accepted division (Ceder and Wilson, 1986) considers (at least) the stages of route design, frequency setting, timetable construction, fleet and crew assignment.

In this work, we consider the problem of designing the number and itinerary of bus routes and their corresponding frequencies, in the context of the strategic (i.e. long term) planning of a public transportation system. The routes should be defined in terms of a given infrastructure of streets and stops and should cover a given origin–destination demand; the complete determination of frequencies is a tactical planning problem, however they are subject to a preliminary setting at the strategic level. The problem of determining routes and frequencies has many variants (Baaj and Mahmassani, 1991; Borndörfer et al., 2007; Fernández et al., 2008; Zhao and Dessouky, 2008) and designations. Here, according to the (strategic) planning context (Desaulniers and Hickman, 2007), we assume that in general terms the problem considers the maximization of the level of service (represented by the minimization of the total travel time of the users), taking into account constraints on the cost of the operators, and other physical and policy constraints. In particular, we explicitly assume that the users are sensitive to the waiting time and to transfers. This fact has strong implications in our modeling, which excludes other aspects of the problem, like the interaction with other transportation modes, the elasticity of the demand and the fares. To refer to this problem, we use the terminology TNDP: Transit Network Design Problem (Baaj and Mahmassani, 1991).

The TNDP is modeled in terms of graphs, whose nodes represent intersections of streets or zone centroids and whose arcs represent connections between such nodes (a street segment between two consecutive intersections or a connection between two adjacent zones). TNDP can be considered as a complex variant of the NP-hard combinatorial problem of network design (Magnanti and Wong, 1984); but in addition to determining which arcs to include in the solution, one should determine how these arcs are combined to form different routes and their frequencies (Desaulniers and Hickman, 2007). Also, one needs to model the behavior of the users with respect to a set of routes and frequencies, through a so called *assignment sub-model*. Such a model distributes the origin–destination demand among the routes and it is necessary in order to compute measures like the travel time of the users and the occupancy level of the buses.

The TNDP has been tackled in the literature by means of three different approaches: (a) heuristics without an explicit mathematical formulation (Baaj and Mahmassani, 1991), mainly based on constructive and improvement procedures, (b) metaheuristics (Pattnaik et al., 1998), including different mechanisms for exploring the search space, and (c) explicit mathematical programming formulations (Borndörfer et al., 2007). Up to now, the heuristics and metaheuristics approaches have proved to be the only practical methods to solve realistic and large-sized problem instances (Bagloee and Ceder, 2011; Cipriani et al., 2012; Fernández et al., 2008; Zhao and Zeng, 2008). Mathematical programming approaches face the difficulties of modeling the different aspects of the problem (e.g. user behavior and route structure) and the computational complexity of solving the resulting model. On the other hand, mathematical programming approaches enable to analyze in a precise way the interactions among the different components of the system, thus allowing to gain insight on the problem structure and potential solving methods. In this work, we advance in the modeling side of TNDP by including into a single mathematical programming formulation, several aspects of the problem which have not been put together in the existing literature, namely: (a) the interest of the users, (b) the interest of the operators, (c) the behavior of the users (specially by taking into account the effect of waiting time and multiple lines), and (d) constraints regarding transfer, infrastructure and bus capacity. Through an implementation of the formulation which considers (a), (b) and (c), we investigate experimentally the numerical solution of the model applied to different problem instances, including cases existing on the literature and a case based on the transport network of a real city in Uruguay. We also present and discuss an extension of the model to include aspect (d), which results in a bi-level version of the formulation.

The article is structured as follows. Section 2 presents the basic concepts, terminology and notation. Section 3 reviews the existing mathematical programming formulations in light of the problem aspects considered as relevant to have a realistic model, while Sections 4 and 5 present the proposed formulation which considers those aspects. Section 6 reports numerical results obtained by running the proposed model on different case studies and Section 7 elaborates conclusions and proposes future work.

## 2. Basic concepts, terminology and notation

In this section we present the basic concepts, definitions and part of the notation used in the remaining part of the article. Firstly, we present the graph model used to represent the elements of a public transportation system which are relevant to the TNDP. After, we present the concepts related to the assignment sub-model. Finally, we explain the more common approaches to model several basic components of TNDP, which are used in the different models reviewed and proposed in this work.

### *2.1. Graph model*

The TNDP requires to represent the infrastructure over which the routes will be defined and the demand that should be satisfied. Moreover, once the routes are defined, it is necessary to represent the trajectories that the users will follow from their origins to their destinations using those routes.

The infrastructure is given by the streets of the city which are enabled to run transit vehicles and the stops of the system. The demand is represented by centroids, which are fictitious points that concentrate the entire demand of a zone