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Route choice and traffic signal control: A study of the stability and instability of a new dynamical model of route choice and traffic signal control

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ABSTRACT

This paper presents a novel idealised dynamical model of day to day traffic re-routeing (as traffic seeks cheaper routes) and proves a stability result for this dynamical model. (The dynamical model is based on swapping flow between paired alternative segments (these were introduced by Bar-Gera (2010)) rather than between routes.) It is shown that under certain conditions the dynamical system enters a given connected set of approximate equilibria in a finite number of days or steps. This proof allows for saturation flows which act as potentially active flow constraints. The dynamical system involving paired alternative segment swaps is then combined with a novel green-time-swapping rule; this rule swaps green-time toward more pressurised signal stages. It is shown that if (i) the delay formulae have a simple form and (ii) the "pressure" formula fits the special control policy P_0 (see Smith, 1979a,b), then the combined flow-swapping/green-time-swapping dynamical model also enters a given connected set of approximate consistent equilibria in a finite number of steps. Computational results confirm, in a simple network, the positive P_0 result and also show, on the other hand, that such good behaviour may not arise if the equisaturation control policy is utilised. The dynamical models described here do not represent blocking back effects.

1. Introduction

1.1. A brief route choice and traffic control modelling context

Dynamic transport models involving both travellers' choices (including drivers' repeated route choices) and traffic signal controls are needed. Such models may be used

(i) *to help predict* (for a given responsive control strategy) how traffic flows and controls are likely to evolve over time and so to help assess different given control strategies (against specified congestion, delay, pollution, accessibility or other criteria); and

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(ii) *to help design* new control strategies for reducing congestion, delay, pollution, inaccessibility (or other criteria) in cities, taking reasonable account of the future evolution of traffic flows as these respond to the control strategies.

Allsop (1974), Gartner (1976), Smith (1979a,c), Bentley and Lambe (1980) and Dickson (1981) were among the first to point to the need to combine models of route choice and traffic signal control; in part so that optimal controls taking account of routeing reactions might be found. The study of traffic control and route choice has been pursued by Meneguzzer (1996, 1997), Maher et al. (2001), Wong et al. (2001), and many others. Taale and van Zuylen (2001) provide an overview.

Cantarella et al. (1991) and Cantarella (2010) focus on seeking optimal controls which take account of route choices. They address stability issues involving both routeing and control. In these papers, a bi-level optimisation method is used as the signal setting method. The route choice model used finds for each OD pair a cheapest route and then swaps route flow toward the cheapest route.

In this paper we consider a joint, two-commodity (route flow, green time) dynamical system; in which route-flows switch toward cheaper routes and signal green-times switch to more pressured stages. Both route-flow and green-time swaps follow a development of the 'proportional adjustment process' dynamical system in Smith (1984a).

In combined traffic signal control and route-choice models we consider not only costs of routes (which will causes route flows to change) but also "pressures" on signal stages (which will cause stage green-times to change). This formulation was perhaps first introduced in Smith et al. (1987) and Smith (1987). In the models here both route costs and stage pressures will be functions of flows and green-times. These given functions determine (flow, green-time) pairs which satisfy Wardrop's equilibrium condition and a specific control policy as follows:

A (route-flow, green-time) pair satisfies the Wardrop equilibrium condition if:

$$\text{more costly routes carry no flow.} \tag{1.1}$$

A (route-flow, green-time) pair satisfies the signal control policy if:

$$\text{less pressurised stages receive no green-time.} \tag{1.2}$$

Condition (1.1) holds if and only if Wardrop's equilibrium condition is exactly satisfied. If (1.1) does not hold exactly then it is proposed initially in this paper that for each pair of routes joining each OD pair, route flow swaps from the more costly route to the less costly route at a rate which is proportional to:

$$(\text{the difference in the route costs}) \times (\text{flow along theroute with the greater cost}).$$

Similarly, condition (1.2) holds if and only if the control policy is exactly satisfied. If (1.2) does not hold exactly then, in this paper, for each pair of stages at each junction the stage green-time swaps from the less pressurised stage to the more pressurised stage at a rate which is proportional to:

$$(\text{the difference in the stage pressures}) \times (\text{green-time given to the stage with the smaller pressure}).$$

The stability of this combined routeing and signal-control dynamical system is considered in this paper. Smith and Mounce (2011) have considered a restricted form of this dynamical model within a very different context: that of splitting rates.

A wide-ranging route choice and signal control modelling context is given in Appendix A.

1.2. Overview and contributions of this paper

This paper focuses on certain mathematical models of *route choice dynamics* and combined *route-choice and traffic signal control dynamics* and considers the *stability* of these dynamical models.

The routeing plus signal control dynamical system represents car drivers seeking better routes and signal timing changes in response to changing traffic flows. The combined (routeing, signal control) dynamical model is idealised. The signal control model may be regarded as a model of a system periodically updated either by an operator or by an automatic system. The dynamical routeing model is designed to approximately represent, albeit in a simplified or idealised form, how routeing decisions are actually made day after day.

The first contribution of this paper is to introduce a new route choice dynamical system; this is a restricted version of the proportional-switch adjustment process (or *PAP*) suggested in Smith (1984a) and discussed by He et al. (2010). He et al. (2010) show that the *PAP* route-swapping model is not always realistic. In this paper we put forward a *restricted proportional adjustment process (RPAP)* to take account of the comments by He et al. (2010) while maintaining the essential (proportional) characteristics of the *PAP*. Paired alternative segments (introduced by Bar-Gera (2010)) form a central element of a *RPAP*.

Other route swap algorithms have been considered by Cascetta (1989), Smith and Wisten (1995), Bellei et al. (2005), Huang and Lam (2002), Peeta and Yang (2003), Nie and Zhang (2005), Nie (2010), Mounce (2006, 2009), Mounce and Carey (2011) and Mounce and Smith (2007). None utilise paired alternative segments.

Secondly, we show that the above route-swapping dynamical system satisfies a stability property similar to that already proved for the more artificial *PAP* dynamical re-routeing system described in Smith (1984a). To be precise it is shown that, under natural conditions, a trajectory of the route-flow dynamical system enters a set of approximate equilibria in a finite number of "days". This routeing stability is guaranteed using *RPAP* and a discrete dynamical system with fixed step lengths