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Continuum signalized junction model for dynamic traffic networks: Offset, spillback, and multiple signal phases

Ke Han [a,*], Vikash V. Gayah [b]

[a] Department of Civil and Environmental Engineering, Imperial College London, United Kingdom
[b] Department of Civil and Environmental Engineering, Pennsylvania State University, USA

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ABSTRACT

This paper extends the continuum signalized intersection model exhaustively studied in Han et al. (2014) to more accurately account for three realistic complications: signal offsets, queue spillbacks, and complex signal phasing schemes. The model extensions are derived theoretically based on signal cycle, green split, and offset, and are shown to approximate well traffic operations at signalized intersections treated using the traditional (and more realistic) on-and-off model. We propose a *generalized continuum signal model*, which explicitly handles complex vehicle spillback patterns on signalized networks with provable error estimates. Under mild conditions, the errors are small and bounded by fixed values that do not grow with time. Overall, this represents a significant improvement over the original continuum model, which had errors that grew quickly with time in the presence of any queue spillbacks and for which errors were not explicitly derived for different offset cases. Thus, the new model is able to more accurately approximate traffic dynamics in large networks with multiple signals under more realistic conditions. We also qualitatively describe how this new model can be applied to several realistic intersection configurations that might be encountered in typical urban networks. These include intersections with multiple entry and exit links, complex signal phasing, all-red times, and the presence of dedicated turning lanes. Numerical tests of the models show remarkable consistency with the on-and-off model, as expected from the theory, with the added benefit of significant computational savings and higher signal control resolution when using the continuum model.

1. Introduction

Signalized intersections are typically the most restrictive bottlenecks in urban environments. For this reason, the performance of individual arterials or small networks is usually described by the operations at the signalized intersections that make up these facilities (TRB, 2000). These locations also tend to be the primary focus of local and network-wide urban traffic control strategies (e.g., see Miller, 1963; Robertson and Bretherton, 1974; Shelby, 2004; Chitour and Piccoli, 2005; Gayah and Daganzo, 2012; Guler and Cassidy, 2012; Gu et al., 2014). Therefore, accurate models of traffic dynamics at individual signalized intersections are essential to the design, management and control of large urban transportation networks.

* Corresponding author.
E-mail addresses: k.han@imperial.ac.uk (K. Han), gayah@engr.psu.edu (V.V. Gayah).

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Unfortunately, operations at signalized intersections are complicated by the cyclical on-and-off pattern of traffic signals. While the pattern is conceptually simple – vehicles are allowed to move through the signal during the green phase or 'on' period and are restricted from moving during the red phase or 'off' period – the periodic switching between these two distinct phases must be tracked to accurately model vehicle throughput. Binary variables are typically used to track the phase of the traffic signal assigned to each approach of every signalized intersection. However, incorporating many binary variables into the optimization frameworks used to study large-scale urban networks (such as Improta and Cantarella (1984) and Lo (1999a,b)) results in complex mixed integer mathematical programs (MIMPs) that are difficult to solve exactly. Instead, heuristic optimization methods must be used (Foy et al., 1992; Chiu and Chand, 1993; Ceylan and Bell, 2004; Murat and Gedizlioglu, 2005), but these provide inexact and suboptimal solutions. Even when exact methods are available to solve MIMPs, they are typically time intensive and computational expensive. The discrete on-and-off nature of traffic signals also results in discontinuous travel time functions with discrete jumps that make dynamic traffic assignment models (Friesz et al., 1993, 2013) difficult to implement. This means that the combined treatment of dynamic traffic assignment with signal control (DUESC) becomes especially difficult to solve (Aziz and Ukkusuri, 2012; Ukkusuri et al., 2013).

To overcome these challenges, recent studies (Smith, 2010; Ge and Zhou, 2012) have proposed a continuum model to approximate traffic dynamics at a signalized intersection without the need for distinct signal phases. A fraction, η, of the downstream link's capacity is assumed to be available to vehicles discharging from a given approach during the entire signal cycle. In this simple model, η, is assumed to be equal to the proportion of the cycle allocated to the subject approach for movement through the intersection. This continuum model has a number of advantages over the on-and-off approach: (1) it requires fewer integer variables when modeling dynamics on large-scale networks, which decreases the computational complexity of modeling and optimization processes; (2) it provides more flexibility in selecting the time resolution in a discrete-time environment; (3) it eliminates discontinuities in travel time functions, which allows dynamic user equilibrium problems (Friesz et al., 2013) to be formulated in more exact ways without the need to introduce indifference behaviors (Szeto and Lo, 2006; Ge and Zhou, 2012; Han, 2013); and (4) it imposes fewer implicit constraints on potential signal timings in optimization procedures. More details on these advantages are provided in Han et al. (2014). However, the primary drawback of this approach is that it does not exactly replicate the dynamics that would arise at an intersection. If significant errors exist between the two approaches, the continuum representation becomes invalid. This model appears to be an extension of the 'store-and-forward' model (SFM), originally proposed by Gazis and Potts (1963), which assumes flows discharge continuously through an intersection movement at a rate equal to the product of the green ratio and saturation flow. The SFM is restrictive in that it assumes that the signal is always completely saturated. Aboudolas et al. (2009) extended the SFM to consider under-saturated conditions and complete flow blockages from downstream queues. However, this model is still deficient as it fails to account for partial disruptions that might occur due to spillback with queued flows greater than zero. Furthermore, it cannot accommodate cycle lengths and offsets as variables in signal timing optimization procedures.

Recently, Han et al. (2014) performed a comprehensive comparison between the continuum model (Smith, 2010; Ge and Zhou, 2012) and the on-and-off signal model. By applying the Hamilton–Jacobi formulation and generalized Lax–Hopf formula (Aubin et al., 2008; Claudel and Bayen, 2010) to the Lighthill–Whitham–Richards model of traffic on links (Lighthill and Whitham, 1955; Richards, 1956), the authors were able to quantify the maximum errors that arise within the continuum model when compared with the exact on-and-off model. This study found that the continuum model works quite well in the absence of queue spillback to the subject intersection. In this case, errors in predicted vehicle counts between this approximation and the exact on–off model are bounded and quite small. However, the approximation does not perform well when queue spillback occurs. For sustained spillback that lasts multiple cycles, errors are large and grow with time. The magnitude of these errors are generally smaller when using a strictly concave fundamental diagram as opposed to a triangular or trapezoidal fundamental diagram, although those errors still grow with time and are unbounded. Moreover, the continuum model does not accurately approximate cases with transient spillbacks that occur entirely during the duration of a signal's cycle. Both sustained and transient spillback frequently arises in congested urban networks,[1] and these limitations reduce the applicability of the continuum signal model. The continuum model studied in that paper also fails to account for offsets between adjacent signals. These offsets significantly impact vehicle travel times along a long arterial that contains many signalized intersections. Furthermore, the continuum model was only proposed for fairly simplistic intersection configurations.

In light of this, the purpose of this study is to extend the simple continuum signal model studied in Han et al. (2014) to account for these more realistic complications. Specifically, a modification of the original continuum model is theoretically derived to accommodate signal timing offsets between adjacent signals along a corridor, assuming the absence of vehicle spillback. Moreover, a *generalized continuum signal model* (GCSM) is proposed to handle the presence of both sustained and transient queue spillbacks in the most general setting, which also takes into account signal offsets. Notably, the GCSM subsumes the Han et al. (2014) model as a special case and is theoretically proven to accurately approximate complex dynamics on signalized networks regardless of the presence of spillbacks. It can even handle spillbacks that affect multiple junctions at the same time, which is not considered in any existing continuum signal model. Qualitative discussions are also provided to demonstrate how the model can be applied to realistic intersection configurations that have multiple entry or exit links, complex signal phasing schemes, all-red times and dedicated turn-lanes. Using these extensions, the new continuum model can be used to study more realistic networks with increased accuracy, while maintaining many of the benefits of

[1] A concrete example of the sustained and transient spillbacks is presented in Section 4.1 of this paper.