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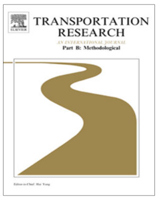

A new schedule-based transit assignment model with travel strategies and supply uncertainties

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This paper proposes a new scheduled-based transit assignment model. Unlike other schedule-based models in the literature, we consider supply uncertainties and assume that users adopt strategies to travel from their origins to their destinations. We present an analytical formulation to ensure that on-board passengers continuing to the next stop have priority and waiting passengers are loaded on a first-come-first-serve basis. We propose an analytical model that captures the stochastic nature of the transit schedules and in-vehicle travel times due to road conditions, incidents, or adverse weather. We adopt a mean variance approach that can consider the covariance of travel time between links in a space–time graph but still lead to a robust transit network loading procedure when optimal strategies are adopted. The proposed model is formulated as a user equilibrium problem and solved by an MSA-type algorithm. Numerical results are reported to show the effects of supply uncertainties on the travel strategies and departure times of passengers.

1. Introduction

A transit assignment model is useful in estimating or predicting how passengers utilize a given transit system. In the literature of transit assignment studies, these models used either the frequency-based (static) or the schedule-based (dynamic) approach to model transit route choice. Similar to the traditional static user equilibrium assignment models, frequency-based transit assignment models (Spiess and Florian, 1989; De Cea and Fernandez, 1993; Cantarella, 1997; Lam et al., 1999, 2002; Kurauchi et al., 2003; Cepeda et al., 2006; Schmöcker et al., 2009, 2011; Sumalee et al., 2009; Cortés et al., 2013; Trozzi et al., 2013; Szeto and Jiang, 2014) often assume that passengers select transit routes to minimize their perceived expected travel cost, and departure time is not the concern. These static transit assignment models are commonly adopted for the strategic and long-term planning/evaluation of transit networks.

Schedule-based transit assignment models (Wilson and Nuzzolo, 2004; Poon et al., 2004; Hamdouch and Lawphongpanich, 2008; Hamdouch et al., 2011; Zhang et al., 2010; Nuzzolo et al., 2012) are typically dynamic and are better suited to short-term transit operations and service planning such as transit timetabling and vehicle scheduling. In a schedule-based model, the temporal dimension is the most important part as it is assumed that transit passengers choose not only their transit routes, but also their departure times for minimizing their individual generalized cost. Researchers incorporate this time dependent choice in different ways which is classified by Poon et al. (2004) as (a) diachronic graph representation (Nuzzolo et al., 2001); (b) dual graph representation (Moller-Pedersen, 1999); (c) forward star network

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formulation (Tong and Wong, 1998), and; (d) space–time formulation (Nguyen et al., 2001; Hamdouch and Lawphongpanich, 2008; Hamdouch et al., 2011). In the last representation, the schedule-based transit network is represented by a time-expanded graph. This graph has an explicit representation of single runs and allows a more straightforward treatment of congestion when capacity constraints are considered. Moreover, it can explicitly represent passenger movements through the in-vehicle and waiting links in the space–time network. This representation and the first one both consider space–time nodes and links. However, a time-expanded network is built on a two dimension graph with one time axis and one space axis. A diachronic network is built in a three dimension graph with two space axes and one time axis.

To model the route choice, one commonly approach is to adopt the concept of optimal strategy. In the frequency-based approach, the core idea for an optimal strategy is that a traveler selects, at each node of the network, a set of attractive lines that allows him/her to reach his/her destination at a minimum expected cost (Spiess and Florian, 1989; Wu et al., 1994; Cepeda et al., 2006; Schmöcker et al., 2009). Different from the previous static models, Hamdouch and Lawphongpanich (2008) developed a dynamic schedule-based transit assignment where the choice of strategy is an integral part of user behavior. In that study, passengers specified their individual travel strategy by providing, at each transit station and each point in time, an ordered list of transit lines they preferred to use to continue their own journey. For a given passenger, the user-preference set at each time-expanded (TE) node collectively yielded a set of potential paths that departed from the passenger's origin at the same time and generally arrived at the destination at different times. Also, when loading a transit vehicle at a station, on-board passengers continuing to the next station remained on the vehicle and waiting passengers were loaded in a first-come-first-serve (FCFS) basis. To explicitly consider vehicle capacities, the model assigned the fail-to-board passengers to the wait arc to wait for their next preferred transit services with residual capacities. Hamdouch et al. (2011) extended the model in Hamdouch and Lawphongpanich (2008) to differentiate the discomfort level experienced by the sitting and standing passengers. Each class of passengers, grouped by their remaining journey lengths and times already spent on-board, was assigned success-to-sit, success-to-stand, and failure-to-board probabilities. These probabilities were computed by performing a dynamic network loading. The stimulus of a standing passenger to sit increased with his/her remaining journey length and time already spent on-board. When a vehicle was full, passengers unable to board must wait for the next vehicle to arrive.

The above studies do not consider the effect of the uncertainties of transit networks on route choice. In fact, due to supply side uncertainties, in-vehicle travel times and waiting times, especially for buses and mini-buses, are highly uncertain. Studies such as Jackson and Jucker (1982) and Szeto et al. (2011b) found that travel time uncertainty does affect the route choice of passengers. It is essential to capture this realistic travel behaviour into the transit modelling framework. Therefore, transit assignment models have recently emphasized the influence of uncertainties in the frequency-based framework and their transit network design applications (Yang and Lam, 2006; Li et al., 2008, 2009; Sumalee et al., 2011; Szeto et al., 2011b, 2013) as in traffic assignment (Shao et al., 2006; Szeto et al., 2011a). These transit assignment models can be used to study the aggregated stochastic effects of transit lines from a static perspective. However, uncertainties exist in both the vehicle running and dwelling processes in line operation and the schedule-based models provide means to investigate uncertainties within the vehicle processes (Zhang et al., 2010). Hence, Zhang et al. (2010) developed a schedule-based transit assignment model to capture the uncertainties, wherein they adopted the effective travel cost as the factor affecting the route choice of passengers and considered chance constraint for dealing with the capacity. Nevertheless, they proposed a path-based model and hence path enumeration or column generation is needed to obtain solutions. Optimal strategies and hence the concept of the set of attractive lines are also not explicitly considered in their model.

The objective of the paper is to extend the schedule-based transit assignment model proposed by Hamdouch and Lawphongpanich (2008) to consider supply uncertainties in the transit network, optimal strategies, and hard capacity constraints. This extension is not straightforward, as the resultant problem is a stochastic and dynamic optimization problem. We propose an analytical model that captures the stochastic nature of the transit schedules and in-vehicle travel times due to road conditions, incidents, or adverse weather. We adopt a mean variance approach that can consider the covariance of travel time between links in a space–time graph but still lead to a robust transit network loading procedure when optimal strategies are adopted. We formulate the problem as a user equilibrium problem. We adopt a user equilibrium (UE) framework instead of a stochastic user equilibrium (SUE) framework because of the following:

(i) It is easier to illustrate the concept of travel strategy and the model formulation clearly and analyze the model properties without being smeared by other factors such as the perception error of passengers on travel costs.
(ii) SUE transit assignment models require a probabilistic choice model to depict the travel choice behavior of passengers. However, a realistic choice model always has some limitations. For example, the Probit model used in SUE transit assignment (e.g., Nielsen, 2000 and Nielsen and Frederiksen, 2006) relies on simulation that suffers from computational burden. The Logit model used in transit assignment models (e.g., Lam et al., 1999, 2002) suffer from the path overlapping issue. Solving C-Logit (Cascetta et al., 1996) and other path-based choice models often requires a path set generation or path enumeration algorithm, and an efficient link based algorithm that obviates the path set generation or enumeration procedure has not yet been developed to solve these models.
(iii) A UE framework has a good mathematical property that allows the dynamic programming technique to be used during the solution process. The technique does not rely on path set generation or path enumeration during that process.