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A variational formulation for higher order macroscopic traffic flow models: Numerical investigation

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ABSTRACT

This paper deals with numerical methods providing semi-analytic solutions to a wide class of macroscopic traffic flow models for piecewise affine initial and boundary conditions. In a very recent paper, a variational principle has been proved for models of the Generic Second Order Modeling (GSOM) family, yielding an adequate framework for effective numerical methods. Any model of the GSOM family can be recast into its Lagrangian form as a Hamilton–Jacobi equation (HJ) for which the solution is interpreted as the position of vehicles. This solution can be computed thanks to Lax–Hopf like formulas and a generalization of the inf-morphism property. The efficiency of this computational method is illustrated through a numerical example and finally a discussion about future developments is provided.

1. Introduction

1.1. General background

In order to get a realistic estimation of the real-time traffic states on networks, traffic operators and managers need macroscopic traffic flow models. These models must be simple, robust, allowing to get solutions at a low computational cost. The main macroscopic models are based on conservation laws or hyperbolic systems (see Hoogendoorn and Bovy (2001); Kuhne and Michalopoulos (1997) for traffic aspects and Garavello and Piccoli (2006) for mathematical aspects). The seminal LWR model (for Lighthill–Whitham and Richards) was proposed in Lighthill and Whitham (1955), Richards (1956) as a single conservation law with unknown the vehicle density. This model based on a first order Partial Differential Equation (PDE) is very simple and robust but it fails to recapture some empirical features of traffic. In particular, it does not allow to take into account non-equilibrium traffic states mainly in congested situation. More sophisticated models referred to as *higher order* models were developed to encompass kinematic constraints of real vehicles or also the wide variety of driver behaviors, even at the macroscopic level. In this paper we deal with models of the Generic Second Order Modeling (GSOM) family. Even if these models are more complicated to deal with, they permit to reproduce traffic instabilities (such as the so-called *stop-and-go* waves, the hysteresis phenomenon or capacity drop) which move at the traffic speed and differ from kinematic waves (Zhang, 2002) (see also Lebacque et al. (2007) and references therein).

Before the wide propagation of internet handsets, traffic monitoring has mainly been built on dedicated infrastructure which imply quite important installation and maintenance costs. Traffic flow monitoring and management has been deeply

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modified with the development of new technologies in mobile sensing aiming to provide a quite important quantity of floating car data. Traffic flow models are needed to be well suited such that managers could use both Eulerian and Lagrangian data for improving traffic state estimation. The term *Eulerian* refers to "classical" fixed equipment giving records of occupancy or flow of vehicles on a freeway section. This kind of measurements come from e.g. fixed inductive loop detectors, Radio Frequency Identification (RFID) transponders, radars or video cameras. By opposite, the term *Lagrangian* is used to characterize data coming from sensors which move within the measured field of interest. Lagrangian data are provided by on board mobile sensors such as *Global Positioning Systems* (GPS) or GPS-enabled *smartphones*, without being exhaustive.

While the idea of monitoring traffic using mobile sensors appeared less than ten years ago with the popularization of the mobile internet and cellular devices, there exists a fast growing literature about how to integrate Lagrangian data into classical macroscopic traffic flow models. The process of incorporating Eulerian and Lagrangian data into a mathematical model to improve the modeling is called *data estimation* or equivalently *inverse modeling*. According to the major UC Berkeley field experiment named *Mobile Century* and then *Mobile Millennium* investigating Lagrangian sensing with GPS-enabled smartphones, it has been shown that even a 2% to 5% penetration rate of probe vehicles into the driver population provides sufficient and accurate data for estimating traffic velocity or density on highways (Herrera et al., 2010; Herrera and Bayen, 2010; Work et al., 2009). Nevertheless, it has been demonstrated in Piccoli et al. (2012) that the quality of estimation for *higher-order traffic quantities* including vehicle acceleration/deceleration, emission and fuel consumption rates is dramatically affected when the penetration rate of probe vehicles or the sampling frequency of the current mobile sensors decrease. However on board devices propose a real breakthrough in traffic monitoring by providing a very cheap and efficient way to collect traffic data.

1.2. Motivation

In order to improve traffic states estimation from Lagrangian data, we propose to deal with macroscopic traffic flow models of the GSOM family. As these models combine the simplicity of the LWR model with the dynamics of driver specific attributes, we are able to recapture more specific phenomenon with a higher accuracy. While methods of data assimilation have been only developed for first order models up to now, this work presents a new algorithm to reconstruct traffic states from both Eulerian and Lagrangian data. We take advantage of a very recent article (Lebacque and Khoshyaran, 2013) in which a variational principle has been proved for models of the GSOM family.

1.3. Organization of the paper

The rest of this paper is structured as follows. Section 2 presents more in detail the GSOM models and sheds a specific light on the LWR model which is widely used in traffic engineering. The variational principles for the GSOM models including LWR model are briefly recalled in Section 3. Section 4 is devoted to the presentation of the main elements of our computational method. Finally, Section 4.4 proposes a numerical example of our method.

2. GSOM traffic flow models

2.1. Formulation of GSOM models

In Lebacque et al. (2005, 2007), the authors introduce a general class of macroscopic traffic flow models called the Generic Second Order Models (GSOM) family. Any model of the GSOM family can be stated in conservation form as follows

$$\begin{cases} \partial_t \rho + \partial_x(\rho v) = 0 & \text{Conservation of vehicles,} \\ \partial_t(\rho I) + \partial_x(\rho v I) = \rho \varphi(I) & \text{Dynamics of the driver attribute } I, \\ v = \mathfrak{I}(\rho, I) & \text{Fundamental diagram,} \end{cases} \tag{2.1}$$

where ρ stands for the density of vehicles, v for the flow speed (equal to the mean spatial velocity of vehicles), x and t for position and time. The variable I is a specific driver attribute which can represent for example the driver aggressiveness, the driver destination or the vehicle class. The flow-density fundamental diagram (FD) is defined by

$$\mathfrak{F} : (\rho, I) \mapsto \rho \mathfrak{I}(\rho, I).$$

Notice moreover that it was shown in Lebacque et al. (2007) that the notions of Supply and Demand functions defined in Lebacque (1996) for the classical LWR model could be extended to the GSOM family. The GSOM models admit two kinds of waves:

- Kinematic waves or 1-waves as in the seminal LWR model: a wave propagates density variations at speed $v_1 = \partial_\rho \mathfrak{F}(\rho, I)$ while the driver attribute I is continuous across such a wave.
- Contact discontinuities or 2-waves: a wave propagates variations of driver attribute I at speed $v_2 = \mathfrak{I}(\rho, I)$ while the flow speed is constant across such a wave.