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The shared-taxi problem: Formulation and solution methods

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ABSTRACT

With the rising fuel costs, ride sharing is becoming a common mode of transportation. Sharing taxis which has been prominent in several developing countries is also becoming common in several cities around the world. Sharing taxis presents several advantages as it minimizes vacant seats in cars thus reducing costs on taxi operators which results in significantly lower taxi fares for passengers. Besides the economical advantages, taxi sharing is highly important for reducing congestion on the roads and for minimizing the impact of transportation on the environment. In this paper, we formulate the problem of assigning passengers to taxis and computing the optimal routes of taxis as a mixed integer program. To solve the proposed model, we present a Lagrangian decomposition approach which exploits the structure of the problem leading to smaller problems that are solved separately. Furthermore, we propose two heuristics that are used to obtain good quality feasible solutions. The Lagrangian approach along with the heuristics are implemented and compared to solving the full problem using CPLEX. The computational results indicate the efficiency of the methodology in providing tighter bounds than CPLEX in shorter computational time.

1. Introduction

Inefficiencies in transportation are nowadays causing several economical and environmental problems mainly due to the high levels of traffic congestion resulting in increasingly wasted resources and staggering amounts of pollution. Ride-sharing, which basically aims at minimizing the number of vacant seats in vehicles and therefore reducing the number of required vehicles, emerges as an essential practice to reduce traffic and fuel costs as well as to support the green initiatives that aim at improving the environment.

Several variants of the ride-sharing problem exist, most of which having to do with either developing efficient car pooling systems or optimizing the operation of public transportation. The recent survey of Furuhata et al. (2013) provided an extensive overview of the state of the art ride-sharing systems and discussed the existing challenges. Among the main challenges that have been identified are those that relate to service pricing and passengers matching. The survey of Agatz et al. (2012) focused on the recent research in the area of optimization models for ride-sharing particularly passenger matching and concluded that the field of transportation is far from mature and still has room for major contributions. A problem in which customers request rides from specific pickup locations to specific dropoff locations, while imposing certain requirements on the ride (such as the earliest pickup time and latest dropoff time), is known in the literature as the Dial-A-Ride Problem (DARP).

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Several variants of DARP exist including single-vehicle and multi-vehicle DARP as well as static and dynamic DARP. The dynamic version accommodates for changes that include users leaving and entering the system, and most importantly handles new customer requests well after the fleet of riders is spread and already traveling.

The shared-taxi problem falls under the multi-vehicle dynamic DARP, and is defined as follows. Taxi-seekers place a request by indicating pickup locations and their desired dropoff locations. They also indicate the earliest acceptable pickup time and the latest acceptable dropped off time as well as a maximum ride time. The fare that is paid by each passenger is based on the pickup zone and the dropoff zone. This form of taxi sharing has been the main form of public transportation in many developing countries. Very recently, such taxi-sharing methods became common in several cities such as New York, London, and Montreal (Transport Canada, 2011). The shared-taxi problem seeks to determine the optimal assignment of passengers to taxis as well as the optimal route for each taxi.

In this paper, the shared-taxi problem is formulated as a mixed integer program. A Lagrangian decomposition approach that exploits the structure of the problem is proposed as a solution methodology. We also propose a novel heuristic that finds good quality feasible solutions within limited computational time thus making it appropriate for real-time implementations. Extensive computational testing showed that the proposed Lagrangian decomposition outperforms solving the full mixed integer program while the proposed heuristics provide good quality feasible solutions.

The rest of the paper is organized as follows. Section 2 presents a literature review, while Section 3 describes the problem formulation. Section 4 on the other hand presents the Lagrangian decomposition approach, and Section 5 presents a heuristic for finding good feasible solutions. The proposed Lagrangian decomposition approach and heuristics are evaluated in Section 6. Finally, Section 7 concludes and illustrates future research directions.

2. Literature review

The DARP is a generalization of vehicle routing problems, like the Pick-up and Delivery Vehicle Routing Problem and the Vehicle Routing Problem with Time Windows, which have been studied in the literature (Desrosiers et al., 1995; Desaulniers et al., 2002). In Psaraftis (1980) and Psaraftis (1983), a dynamic programming approach, in which the objective function takes into consideration both cost minimization and customer dissatisfaction minimization, was proposed and small instances of the single-vehicle static and dynamic DARP were solved. Cordeau and Laporte (2003) presented a tabu search heuristic for the multi-vehicle static DARP where ride seekers specify time windows on pickups and dropoffs, and drivers serve as many ride seekers as possible starting from a common point and arriving at a final common point. A similar problem where all ride seekers and ride providers are heading to the same destination (employees of the same company heading to office), was solved in Baldacci et al. (2004) using a column generation approach. By nature, such a problem is not dynamic since all inputs are known beforehand and the scheduling can happen as much as days earlier. In contrast, Attanasio et al. (2004) proposes parallel tabu search heuristics for the dynamic DARP where an initial static solution is first achieved and then the new passenger requests are inserted into the solution as they arrive. Other metaheuristics such as the genetic algorithm that is presented in Jorgensen et al. (2006), the variable neighborhood search algorithm of Parragh et al. (2010), the stochastic variable neighborhood search algorithm of Schilde et al. (2011), the granular tabu search algorithm of Kirchler and Wolfler Calvo (2013), and the multicriteria tabu search of Paquette et al. (2013) also proved to be successful in finding high quality solutions. It is worth noting that the approach of Paquette et al. (2013) considers a multicriteria objective which includes cost minimization in addition to quality of service measures that relate to waiting and travel times.

An exact approach for solving the multi-vehicle static DARP was presented in Cordeau (2006) where a branch-and-cut algorithm that uses valid inequalities that are derived from the dial-a-ride, the traveling salesman, the vehicle routing, and the pick-up and delivery problems is used to efficiently solve small to medium-size instances. Based on the 3-index formulation that was presented in Cordeau (2006), Ropke et al. (2007) proposed two new models based on a 2-index formulation and new families of valid inequalities are derived to strengthen the models.

In Coslovich et al. (2006), the dynamic aspect of the DARP is expressed when drivers which are en-route suddenly receive requests at known stops and must make a decision on whether or not to service these requests. The decision is based on whether or not an efficient insertion algorithm succeeds into inserting the request into the already computed route. Parragh et al. (2012) presented extensions of the 3-index of Cordeau (2006) and the set partitioning formulation of Ropke et al. (2007) to include heterogeneous fleet and passengers, where vehicles can have different capacities and passengers can request different modes of transportation. The effective solution approach integrates variable neighborhood search into a column generation algorithm, and high-quality solutions for realistic test instances are presented.

Xiang et al. (2006) and Xiang et al. (2008) presented fast heuristics for the multi-vehicle static and dynamic DARP based on Local search and several diversification and improvement strategies that improve initial solutions. The special application of dynamic DARP to transporting patients between several locations in a hospital campus was also discussed in Beaudry et al. (2010) which presented a new solution approach based on a two-phase heuristic that includes a tabu search algorithm. In Agatz et al. (2011), a dynamic carpooling problem is considered and a model based on rolling horizon is compared to a greedy heuristic using a simulation study based on travel demand data from metropolitan Atlanta. Most recently, Berbeglia et al. (2012) proposed a strategy in which requests are treated one by one upon arrival. A hybrid algorithm