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Pareto efficiency of reliability-based traffic equilibria and risk-taking behavior of travelers

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ABSTRACT

This paper investigates the Pareto efficiency of the various reliability-based traffic equilibria proposed in the literature and the risk-taking behavior of travelers. Reliability indexes such as the percentile travel time (PTT), travel time budget (TTB), mean excess travel time (METT) and the quadratic disutility function (QDF) are examined in terms of the mean and standard deviation (SD) of travel times. The downward sloping mean-SD indifference curve is introduced to geometrically analyze the risk-taking behavior of travelers. Both the diversifying and plunging behaviors of risk-averse travelers are investigated by examining the curvature of the mean-SD indifference curves at traffic equilibria based on the PTT, TTB, METT and QDF. Several specific probability distributions are adopted to elucidate the theoretical results obtained.

1. Introduction

Many empirical studies have shown that travel time uncertainty in transportation networks affects travelers' route choice (Jackson and Jucker, 1982; Lam and Small, 2001; Abdel-Aty et al., 1997, to name but a few). Chen and Zhou (2010) classified the sources of travel time uncertainty from the supply and demand sides. Bad weather, traffic incidents, and the like lead to road capacity variability and thus typically non-recurrent congestion. Demand side variability mainly results from the travelers' departure time, destination and mode choices. Generally speaking, travelers may arrive late and incur heavy penalties due to the variability of travel time. Their route choice behavior and the traffic equilibrium characteristics are thus influenced by this variability.

Two typical methods, the stochastic dominance (Levy, 1992) and the mean-risk analysis (Markowitz, 1987), are frequently used for modeling choice among uncertain prospects. These two methods are also popular in the study of vehicle routing with stochastic travel time. The relationship between the stochastic dominance and the mean-risk approaches was investigated by Levy and Hanoch (1970) and Ogryczak and Ruszczynski (2001).

The stochastic dominance approach is based on an axiomatic model of risk-averse preferences, where decision makers are assumed to possess a utility function and the choice consists in maximizing the expected utility. It has been applied in stochastic routing problems in transportation networks (Miller-Hooks and Mahmassani, 2003; Wu and Nie, 2011). Nonetheless, the approach does not render a simple computational recipe and the exact information on the travel time distribution of each route is hard to acquire.

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The mean-risk approach quantifies the route travel time uncertainty using two summary statistics: the mean which represents the expected travel time and the risk defined as a scalar measure of the variability of travel time. Travelers are assumed to make a route choice according to the two statistics. Pertaining to this approach is the concept of the travel time budget (TTB) or efficient travel time (ETT) in route choice models adopted by Hall (1983) and Lo et al. (2006). The travel time budget is given as the sum of the mean travel time and a safety budget, the latter of which is commonly defined as the product of the standard deviation of travel time and a scalar. Thus, the risk is measured using the standard deviation, and the route choice rule of travelers involves minimizing the linear combination of the mean and risk. The other relevant concept is the on-time arrival probability, which assumes that travelers will try to maximize the probability of completing a trip within a given reference time (Frank, 1969). In this case, the risk is measured by the quantile of the route choice's loss probability, and maximizing the on-time arrival probability is equivalent to minimizing the risk of the selected route. The value of risk of a route is also termed as the percentile travel time (PTT) for a desired on-time arrival probability (Nie and Wu, 2009). An alternative measure of the risk in route choice was proposed by Chen and Zhou (2010) using the mean excess travel time (METT) for a given travel time.

In fact, a number of earlier transportation analyses have adopted the mean and variance of travel time. Mirchandani and Soroush (1987) adopted the quadratic disutility function (QDF) to model risk-taking behavior by considering both probabilistic travel times and variable perception errors. Yin et al. (2004) also used QDF to characterize each route and assumed that all travelers choose their routes to minimize their disutility. Sivakumar and Batta (1994) proposed the variance-constrained shortest route problem. More generally, Sen et al. (2001) proposed a mean–variance dominance to determine the set of optimal routes in stochastic networks. The mean–variance dominant routes must include the QDF minimization and variance-constrained shortest routes. The use of the summary statistics simplifies the analysis, and is amenable to geometric treatment, but may lead to unwarranted conclusions (Yitzhaki, 1982).

In the context of reliability-based traffic equilibrium models, utility or disutility is often constructed using a combination of the attributes (expected travel time, travel time variance and standard deviation) since all travelers are simply assumed to make a tradeoff between the expected travel time and its uncertainty (Mirchandani and Soroush, 1987; Noland et al., 1998; Yin and Ieda, 2001). More recently, based on alternative risk measures and route choice criteria, a number of new traffic equilibrium models with fixed demand have been proposed. Examples include the percentile user equilibrium model, in which all travelers are assumed to choose routes to minimize their own PTT (Nie, 2011); the deterministic and stochastic traffic equilibrium model with a travel time budget, in which all travelers are assumed to minimize their travel time budget for a pre-determined on-time arrival probability (Lo et al., 2006; Shao et al., 2006; Lam et al., 2008); the α-reliable mean excess traffic equilibrium model, in which all travelers are assumed to minimize their travel risk measured by the conditional expectation of the excess travel time for a certain travel time budget (Chen and Zhou, 2010); and the traffic equilibrium model based on the quadratic disutility function (Yin et al., 2004).

The abovementioned reliability-based traffic equilibrium models in general give rise to different route flow patterns since travelers are assumed to vary in their risk-taking behaviors. In contrast, this paper introduces the concept of a Pareto-efficient route flow pattern in which all used/chosen routes are non-dominated in the sense of the mean and standard deviation of travel time. Based on the reliability indexes such as the PTT, TTB, METT and QDF, we examine the Pareto efficiency of the resulting traffic equilibria for a given travel demand. Here, we are not claiming that Pareto optimality per se should serve as the norm of travelers' route choice under travel time uncertainty, because some travelers would simply ignore the standard deviation of travel time when they make their route choice. Nonetheless, the notion of Pareto efficiency or Pareto optimality is widely used in the study of the economic efficiency of resource allocation (Varian, 1992), in the context of multi-objective optimization in mathematical programming (Ehrgott, 2005), and recently in the analysis and modeling of transportation systems (Guo and Yang, 2009, 2010; Tan et al., 2010; Tan and Yang, 2012; Chen and Yang, 2012; Wang and Ehrgott, 2013). Pareto efficiency should thus be regarded as an important property that reliability-based traffic equilibria should possess, no matter whether the equilibrium model is constructed using the expected utility theory, non-expected-utility theory (e.g., prospect theory considered by Connors and Sumalee, 2009 and Xu et al., 2011), or a heuristic rule such as PTT, TTB and METT mentioned above. Otherwise, at equilibrium some travelers can reduce either the means or the standard deviations of their travel times or reduce both by switching to an alternative route.

Apart from conducting a Pareto efficiency analysis, we also develop a geometrical analysis of the travelers' risk-taking behavior by using the mean-standard deviation (abbreviated as ES) indifference curve, one of the key approaches in portfolio selection (Tobin, 1958). The risk-averting, risk-neutral and risk-loving behaviors pertain to the directions of the ES indifference curve in the ES space. Tobin (1958) further classified the risk-averters into diversifiers and plungers—the former requires a greater compensation per unit risk, while the latter accepts more risk per unit return. The diversifying and plunging behaviors depend on the curvature of the ES indifference curve. We investigate the diversifying and plunging behaviors of the risk-averse travelers at traffic equilibria by examining the curvature of the ES indifference curves associated with the TTB, PTT, METT and QDF.

The rest of the paper is organized as follows. The next section introduces the basic definitions of the mean–variance non-dominated route and the Pareto-efficient route flow patterns. Section 3 examines the Pareto efficiency of the equilibrium route flow patterns derived from the reliability-based traffic equilibrium models mentioned above. Section 4 looks into the curvature of the ES indifference curves and delineates the resultant diversifying and plunging behaviors at traffic equilibria. Specific analysis with several typical probability distributions is conducted in Section 5 and a numerical example is provided in Section 6. Finally, conclusions together with recommendations for future research are given in Section 7.