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Consistent formulation of network equilibrium with stochastic flows

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ABSTRACT

Traffic flows in real-life transportation systems vary on a daily basis. According to traffic flow theory, such variability should induce a similar variability in travel times, but this "internal consistency" is generally not captured by existing network equilibrium models. We present an internally-consistent network equilibrium approach, which considers two potential sources of flow variability: (i) daily variation in route choice and (ii) daily variation in origin–destination demand. We particularly aspire to a flexible formulation that permits alternative statistical assumptions, which allows the best fit to be made to observed variability data in particular applications. Joint probability distributions of route—and therefore link—flows are derived under several assumptions concerning stochastic driver behavior. A stochastic network equilibrium model with stochastic demands and route choices is formulated as a fixed point problem. We explore limiting cases which allow an equivalent convex optimization problem to be defined, and finally apply this method to a real-life network of Kanazawa City, Japan.

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1. Introduction, review and general framework

In recent years, topics under the broad heading of "network reliability" have received an increasing share of research attention. A considerable body of work now exists on explanatory models that relate traveler behavior (especially route choice) to variation in service levels offered by the available alternatives (e.g. travel time variation), or to the inconvenient consequences of that variation (e.g., arriving late at a destination). Mirchandani and Soroush (1987) proposed an extension to the well-known Stochastic User Equilibrium (SUE) model, in which the actual travel times are random in addition to travelers' perceptions of them. To analyze the effect of traffic information, Arnott et al. (1991) introduced random capacity into network equilibrium, whereby informed users were aware of the variations while uninformed users based their routing decisions on long-term expectations. Chen et al. (2002) formulated the "capacity reliability" concept, considering the probability that a network can serve a given level of demand, given stochastic variations in the link capacities (subsequently, also with stochastic variation in the demands themselves). Lo and Tung (2003) and Lo et al. (2006) formulated a probabilistic user equilibrium model under link capacity variations.

More recently, Nie (2011) proposed a percentile user equilibrium model based on random variations in capacity. Yin and Ieda (2001), Yin et al. (2004) and Watling (2006) developed network equilibrium models under the assumption of exogenously-specified travel time distributions. Chen and Zhou (2010), assuming an exogenously-specified lognormal travel time

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distribution, proposed a model in which travelers aim to minimize their "mean-excess travel time". Several authors have explicitly considered the impact of stochastic supply and demand on network equilibrium, including Shao et al. (2006), Siu and Lo (2008), and the studies of adverse weather by Lam et al. (2008) and Sumalee et al. (2010). On a different, but related, theme of network robustness, Waller et al. (2001) and Waller and Ziliaskopoulos (2006) investigated how a planner's uncertainty in the mean demand level affects errors in equilibrium traffic forecasts, and Zhang et al. (2011) introduced the concept of expected residual minimization into stochastic-flow network equilibrium.

Clearly there have been many developments to the array of tools available for the analysis of stochastic networks. The purpose of this paper is to highlight an issue that, to some degree, is common to any such method of analysis, namely that of the *internal consistency* between the assumptions made regarding stochastic variation of various components of the traffic system, and in particular how this may be resolved within the context of an equilibrium approach. One approach adopted in several reliability/robustness analyses is to view the variability as external to the equilibrium process, in that the approach generates random input data to which some conventional notion of equilibrium is applied. As argued in Clark and Watling (2005), such an approach seems less appropriate for studying network unreliability due to day-to-day variability, since it is unlikely that the travelers in the transport system will be able to equilibrate on a daily basis. In this paper, on the other hand, we consider what 'equilibrium' might mean in a daily varying system. While we could consider more complex model forms and sets of assumptions, our focus will be on a relatively 'stripped down' class of models, in which we return to the basic foundation of SUE, and explore how we might formulate it in a coherent way when we may have stochastic, day-to-day variation (a) in the OD demands and/or (b) in the route choices given any demands.

Consistently incorporating the resulting distributions of both route flow and route travel time into network equilibrium is a non-trivial problem. Consider, as a starting point, the conventional SUE model, in which route utility is a sum of a systematic part (typically, the mean travel time) and a random residual term. Although the random term introduces a stochastic element, the flows in the SUE model are regarded as deterministic. However, once the equilibrium route choice proportions have been computed, a probability distribution of route flows between each origin–destination (OD) pair could be derived, *ex post facto*, as a multinomial distribution (Sheffi, 1985, p. 281), which could then be combined to generate a probability distribution of link flows and thence link travel times. However, for a non-linear travel time function $t(x)$, this will induce an inconsistency; for example, since under a random flow X, it is the case that $\mathrm{E}[t(X)] \neq t(\mathrm{E}[X])$ (see Watling, 2002a), it follows that the mean travel times on which the flow distribution was predicated are not equal to the mean travel times that would arise from a post-analysis of the model.

One approach to addressing this inconsistency is to use Markov processes to model the uncertain, dynamic evolution of networks—see Watling and Cantarella (2013a, 2013b) for recent reviews of this literature. (The Markov process approach is compared, both theoretically and numerically, with network equilibrium models of the kind studied in the present paper in Watling, 2002b.) We do not adopt this approach in the present paper, but rather present an extended formulation of the SUE model that is able to accommodate such variability. The general framework we present is both a synthesis and extension of several existing works in the literature.

A generic description of a network equilibrium mechanism with stochastic flows and travel times is presented in Fig. 1. Since stochastic travel behavior is the main contributor to stochastic network flow, we consider that route choice and/or demand (i.e. whether a traveler makes a trip) could be represented as stochastic variables. Allowing for the possibility of the modeler to represent either route choice or demand as a deterministic or stochastic entity, and given that the case in which both are deterministic is already handled through conventional network equilibrium approaches, there are four important classes of stochastic-flow network equilibrium problems which we shall address: (i) stochastic route choice with deterministic demand, (ii) deterministic route choice with stochastic demand, (iii) stochastic route choice with stochastic demand (or "doubly-stochastic" demand and route choice), and (iv) "compound" stochastic route choice with stochastic (or deterministic) demand.

Table 1 presents existing approaches that fall within our framework. Class i includes the model of Watling (2002a), who addressed the consistency problem between distributions of flow and travel time through a second-order approximation based on multinomial route flows, thus equilibrating the first and second order flow moments (means, variances, and covariances). Class iii includes the extension of this model to include binomial demand variation, as described in Watling (2002c). Also within Class iii, Nakayama and colleagues presented a similar modeling approach (though not requiring any approximation) assuming negative-binomially distributed demand and stochastic route choice (Nakayama and Takayama, 2006; Nakayama, 2007), whereas Clark and Watling (2005, Appendix A) suggested an approach for consistently modeling Poisson variation.

The purpose of this paper is to first bring together, under a common theoretical framework, these previous approaches to consistent modeling of stochastic flows and travel times. In doing so, we aim to highlight a broader range of assumptions that could be adopted within this overall framework; these alternative assumptions are useful when fitting to observed data (some may fit better than others) or because they may have more attractive theoretical properties or be more conducive to efficient large-scale computation. Aside from the general theoretical framing of the problem, our specific technical contributions within this general framework are highlighted in Table 1. A general formulation is presented as a fixed point problem. We subsequently establish the existence of solutions to such models we consider. We also examine limiting or approximate cases, which are appealing as they may be formulated as tractable optimization problems, which is both useful for solution and can be used to establish uniqueness. We conclude by presenting an application of such an approach to the real-life road network of Kanazawa city in Japan.

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